Comments the 2017 NAEP Framework for Mathematics

William McCallum

Shifts in state standards since 2005

The content expectations in the NAEP framework at grades 4 and 8 have remained essentially unchanged since 2005, and at grade 12 since 2006. During that time there has been significant change in state standards, starting with Achieve's *American Diploma Project*, through the 2010 Common Core State Standards in Mathematics (CCSS-M), and continuing today.

From confusion to consensus. In 2005, the distribution of grade levels at which a given topic was addressed across different state standards was extremely wide. For example, the grade in different state standards at which students began to add and subtract fractions ranged from 1 to 7, with solid pluralities in grades 3, 4, and 5 (Reys, 2006). Today, the approximately 40 states that have adopted CCSS-M or similar standards place this expectation at grade 4, the same grade at which this skill is tested by NAEP. In 2006, states had standards on "proportions" ranging from grades 3 to 8 (Reys, 2006). Now there is solid agreement that teaching proportions starts in grade 6 or 7, whereas according to the NAEP framework they are included in the grade 4 assessment (Achieve, 2016). The state consensus has led educators to focus on the most important mathematics for each grade level.

From strands to structure. Most standards in the mid-2000s were organized by strands that spanned all grades from kindergarten to grade 12, such as Number, Measurement, Geometry, and Algebra. This arrangement, allowing for algebra all the way back to kindergarten and number all the way to grade 12, gave license to the mile-wide-inch-deep curriculum in which we "introduce topics early and then repeat them year after year" (Schmidt, Houang, & Cogan, 2002). In contrast, most state standards today follow progressions in which one topic leads to another, with, for example, a focus on arithmetic in grades K–5 leading to a focus on algebra in grades 6–8. Furthermore, standards within a topic are often arranged in conceptually related clusters, which "helps to maintain coherence, ensures that standards are related, and discourages the inclusion of disconnected skills" (Achieve, 2016).

Balance of procedural fluency, conceptual understanding, and applications. During the 1990s and early 2000s, debate raged about which of these three concepts was the appropriate foundation for a sound mathematics education, a debate that contributed to sudden swings in state standards. In its final 2008 report, the National Mathematics Advisory Panel (NMAP) called for an end to this false trichotomy: "To prepare students for Algebra, the curriculum must simultaneously develop conceptual understanding, computational fluency, and problem solving skills. Debates regarding the relative importance of these aspects of mathematical knowledge are misguided" (NMAP, 2008). CCSS- M embraced this balance, which is reflected in state standards to this day.

Implications for NAEP

Because NAEP is constrained by what is actually happening in classrooms, the previous confusion of state standards necessarily showed up in the NAEP assessment framework. The current consensus makes possible a more focused assessment than was possible in 2005. Furthermore, it allows for greater specificity for item developers. Hughes, Daro, Holtzman, and Middleton (2013) noted the lack of specificity in certain areas as a problem.

The shift to more focused and coherent standards has caused some misalignment between NAEP and the states, both in testing things that are not taught, and in not testing things that are taught. For example, the number line, an important tool for understanding fractions, is underemphasized in grade 4 NAEP relative to state standards (Hughes et al., 2013). In grade 8 NAEP, solving systems of linear equations is absent, although it is an important topic at grade 8 in current state standards (Hughes et al., 2015). On the other hand, NAEP, following the strand model, tests many topics inappropriately early, for example, patterns, medians, and proportional relationships in grade 4. For a comprehensive list, see Zimba (2015). As a result of these misalignments, NAEP may not be capturing educational progress accurately.

An important dimension of NAEP is the classification of items into low, medium, and high mathematical complexity. Placing too many topics early could confound this classification. To quote the 2017 NAEP framework, “The demands on thinking that an item expects—what it asks the student to recall, understand, reason about, and do—assume that students are familiar with the mathematics of the task.”

The approach to algebra in NAEP does not reflect the current approach in CCSS-M, and is therefore at odds with standards in most states. Compared to these standards, grade 4 NAEP pays less attention to conceptual basis for algebra in properties of operations; no attention to number line interpretation of fractions or understanding fractions as quantities; and no attention to the role of place value in ordering and comparing whole numbers, or to the importance of attending to the whole in ordering and comparing fractions (Hughes et al., 2013). At grade 8, the balance found in CCSS-M between expressions, equations, and functions is not well reflected in NAEP (Hughes et al., 2013).

Finally, we note that the level of modeling complexity in current state standards for high school is not reflected in grade 12 NAEP.

Recommendations

From the point of view of content alignment there is a clear case for revising the NAEP framework. We recommend:

1. A move away from the strand model to an organization that takes account of the progression of domains in K-12 mathematics and that groups standards in conceptually related clusters. The corresponding change in reporting could give more specific information than currently available, for example on students' skills in multi-digit computation in grade 4.

2. Address obvious topic mismatches as noted in recent alignment studies.

3. Increase the specificity of the framework in areas where overly broad standards provide insufficient guidance to item developers, for example in grade 8 Algebra.

4. Raise the level of modeling complexity in the high school standards.

Appendix: Misconceptions About the Common Core State Standards

Many states have either adopted the Common Core State Standards, or have standards that are very closely modeled on them. Therefore it might be useful to dispel some common misconceptions about the standards. All references are to (NGA & CCSSO, 2010).

1. **It is not true that there are no statistics standards in K-4.** Students collect categorical and measurement data and draw line plots, picture graphs, and bar graphs (see Measurement and Data standards in K-5).
2. **It is not true that CCSS-M deemphasizes procedures.** Indeed, the standards are the first state standards to explicitly require fluency with the standard algorithm for all four arithmetic procedures.
3. **It is not true that the standards discourage calculators in K-8.** The standards neither discourage nor encourage calculators in K-8, leaving policy on that issue as a matter of local control by adopting states. However, Standard for Mathematical Practice #5 (Use Appropriate Tools Strategically), which is on the cover page of every grade level, makes it clear that appropriate opportunities for using calculators should be considered.
4. **It is not true that the standards discourage Algebra in grade 8.** Indeed, much of grade 8 is what used to be in Algebra I: linear equations in one variable, linear functions, and systems of linear equations in two variables. Furthermore, the standards say (p. 84): "The standards themselves do not dictate curriculum, pedagogy, or delivery of content. In particular, states may handle the transition to high school in different ways. For example, many students in the U.S. today take Algebra I in the 8th grade, and in some states this is a requirement. The K-7 standards contain the prerequisites to prepare students for Algebra I by 8th grade, and the standards are designed to permit states to continue existing policies concerning Algebra I in 8th grade."
5. **It is not true that the standards discourage Calculus in grade 12.** The pathway to Calculus is the same under CCSS-M as it was before: get Algebra I, Geometry, and Algebra II finished early enough in your high school career to make room for Calculus.

6. **It is not true that the standards discourage pathways other than calculus**. The significant amount of statistics in the high school standards makes an obvious pathway to AP Statistics in the senior year; the emphasis on modeling in the high school standards prepares students for the senior year modeling class that some states have developed.

References

Achieve. (2016). *Is NAEP math out of step with the states?* Washington, DC: Author.

Hughes, G., Daro, P., Holtzman, D., & Middleton, K. (2013). *A study of the alignment between the NAEP Mathematics Framework and the Common Core State Standards for Mathematics (CCSS-M).* Washington, DC: American Institutes for Research. Retrieved from https://www.air.org/sites/default/files/downloads/report/Study-of-Alignment-NAEP-Mathematics-Items-common-core-Nov-2015.pdf

National Governors Association Center for Best Practices and Council of Chief State School Officers. (2010). *Common Core State Standards for Mathematics.* Retrieved from http://www.corestandards.org/Math/.

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel.* Washington, DC: U.S. Department of Education. Retrieved from https://www2.ed.gov/about/bdscomm/list/mathpanel/report/ nal-report.pdf

Reys, B. (2006). *The intended mathematics curriculum as represented in state-level curriculum standards: Consensus or confusion?* Charlotte, NC: Information Age Publishing.

Schmidt, W., Houang, R., & Cogan, L. (2002). A coherent curriculum: The case of mathematics. *American Educator*, *26*(2), 1–6. Retrieved from https://www.aft.org/sites/default/ les/periodicals/curriculum.pdf

Zimba, J. (2015, October). *Grade 4 and 8 NAEP objectives outside the Common Core*. Retrieved from https://mathedck.wordpress.com/2015/10/15/grade-4-and-8-naep-objectives-outside-the-common-core/.