Updating the NAEP Mathematics Framework

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Since its inception, the mathematics Framework for the National Assessment of Educational Progress (NAEP) has undergone periodic changes, including changes in the substance and number of content categories and changes in the role of processes (Kenney, 2004). Any revision of the NAEP Mathematics Framework at this time must consider the purpose and uses of the framework. Consistent with early visions of the purpose of NAEP (Tyler, 1969), the National Assessment Governing Board (Governing Board) described the NAEP Assessments (the Main NAEP, the State NAEP, and the Long-Term Trend NAEP) as providing “a rich, broad, and deep picture of student mathematics achievement in the United States” (NAGB, 2014, p. 1) and presenting information about both students’ knowledge of mathematics and their ability to apply that knowledge in problem-solving situations. In this *NAEP Mathematics Framework* (2014) document, the Governing Board noted the importance of core mathematical knowledge and skills that form a foundation to the post-secondary lives of U.S. students. This knowledge was said to include “broad competence in mathematical reasoning” (p. 3) and “the ability to integrate and apply mathematics in diverse problem-solving contexts.” (p. 3).

The Governing Board’s recognition of the importance, not only of mathematical *content* but also of mathematical *processes* in the education of U.S. students reflects the decades-long demand for attention in mathematics education to processes, practices, and mathematical habits of mind. This demand parallels the call for mathematics content to include the integration of procedures and concepts (CCSSI, 2010; NRC, 2001). Despite the regular recognition by the Governing Board and its predecessors as well as by the authors of policy documents (CCSSI, 2010; NCTM, 2000; NRC, 2001) of the importance of mathematical thinking or reasoning, problem solving, and conceptual understanding, these goals are notoriously difficult to assess. As a consequence, measures of student achievement such as NAEP have focused more on execution of procedural skills than on measuring conceptual understanding or problem solving.

From the perspective of one who has participated in the 2004 NAEP Mathematics Steering Committee as well as on the Standing Committees on NAEP Mathematics Content, I can suggest several adjustments to the NAEP Mathematics Framework in response to changing foci and increasing technological capacity in U.S. schools. I am not recommending a complete overhaul of the framework, nor am I suggesting the wholesale elimination of objectives within the current areas of mathematical content. For 45 years (Kenney, 2004), some form of the content areas (Number Properties and Operations; Measurement; Geometry; Data Analysis, Statistics, and Probability; and Algebra) have contributed importantly to the NAEP goal of assessing students' understanding of mathematical content. These content areas need to remain and now need to be expanded and supplemented with new areas involving assessment of broader areas of mathematical processes.

Focus Framework on Using Mathematics to Learn About Quantitative Phenomena

It is time to expand emphasis in the NAEP framework from measuring the ability to perform routine mathematical procedures to include measuring the ability to use mathematics to learn about and interact with quantitative phenomena in the world. This ability to interact mathematically with the world requires profound conceptual understanding, the capacity for mathematical processes such as problem solving and mathematical reasoning, and regular access to current technologies.

Capitalize on Technology

Objectives related to conceptual understanding and mathematical processes should assume a more prominent role in the framework, and several attributes of current technological capability[1] can contribute to the potential of the framework for measurement of these objectives. First, universal availability of technological capacity to perform many of the routine mathematical procedures targeted in the NAEP framework suggest a sous-chef (assisting) role for technology in the context of problem solving. Second, current technological capacity allows personally tailored branching of questions targeting mathematical processes at different levels of complexity. Third, the potential for dynamic student-technology interaction in the context of mathematical problems sets a stage that is increasingly amenable to mathematical processes such as mathematical modeling and problem solving. Fourth, the growing availability of AI capacity that can check properties of data sets allows items that require the creation of examples that meet certain criteria (e.g., create a sample of a given number of data for which the median is greater than the mean).

Focus on Key Concepts

Interpreting the results of NAEP beyond the comparison of total scores is notably difficult, partly because the large number of objectives can conflict with the need to assess understanding at a deeper level and to assess students' facility with mathematical processes. One way to temper this conflict might be to identify a few key foundational concepts[2] that cut across NAEP areas of mathematical content (candidates might be function, equivalence, or units) and to expand and deepen the objectives related to those concepts at each grade level (sources might be the Learning Progressions produced in the context of the Common Core work). This strategy might require de-emphasizing a few other (more procedural) objectives.

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[1] The potential for technology in the NAEP assessment has been a constant observation from early in the history of the NAEP Mathematics Assessment (Dossey et al., 1988).
[2] Examples of such key concepts appear in NCTM (2018).

New Attention to Data Analysis and Mathematical Modeling

One NAEP content area in need of expansion is Data Analysis, Statistics, and Probability (with its current subcategories of data representation, characteristics of data sets, experiments and samples, probability, and mathematical reasoning with data[3]). The growing importance of, and increasing technological access to, a data-driven approach to inference suggest the need to re-examine the objectives in this content area.

Moreover, the addition of mathematical modeling to NAEP is needed to be responsive to calls for such goals, as in the GAIMME report (Garfunkel et al., 2016), in school mathematics. Mathematical modeling uses data analysis and problem solving but it is a subset of neither. However, the Data Analysis, Statistics, and Probability strand and perhaps a new Problem Solving strand could include explicit mathematical modeling objectives. Two examples of objectives that might be included are (a) to identify quantities that are relevant to a modeling situation and (b) to understand the relationship between mathematical models and the real-world settings they are intended to represent. Some comments on problem solving in NAEP follow.

Objectives That Target Problem Solving

The ability to use mathematics to learn about quantitative, real-world phenomena requires a facility with problem solving, which requires students to generate solution paths that are new to them. The assessment of problem solving can be problematic in that it couples the need to decide how to generate new (to the problem-solver) responses with the need to call up and execute learned procedures. Objectives that target problem solving (as opposed to objectives requiring only routine mathematical procedures) are needed in the revised NAEP mathematics framework.

[3] Work on specifying essential understandings in statistics (Peck, 2013) suggests that the last subcategory may be better phrased as "statistical reasoning," which is distinguished from mathematical reasoning.

Include Problems That Require Students to Choose and Use Among Available Resources.

One idea for an item type that targets problem solving is to give students a problem and a set of resources (e.g., formulas, procedures, generalizations) some proper subset of which could be used to solve the problem. Solution to the problem would require students not to generate a specific solution but to provide a description of the sequence in which the selected resources could be used to solve the problem. If one of the resources provided is mathematics-specific technology such as a computer algebra system (CAS), a dynamic geometry or statistics tool, or a graphing calculator, this type of item could separate the execution of routine skills from the decision-making involved in problem solving, and, as such, could serve objectives that require emulating problem solving in a technological world.

Include Problem Statements That Include Too Much or Too Little Information

Another problem-solving objective that could better represent solving problems in real-world settings might target solving everyday problems whose descriptions contain too much or too little given information. Students could be asked to generate solutions or to identify (or choose from a list) additional information that would make the total available information sufficient to solve the problem. For example, a problem scenario might address the job of a football stadium manager —a context that could also be leveraged for a mathematical modeling item:

> The main office at a stadium wants to know the total number of minutes, on average, that a fan waits in line in order to get into the football stadium. The team plays 10 games at the stadium each season. The stadium manager knows the average number of minutes that fans waited for each of the 10 games. Determine the overall average number of minutes, or, tell what additional information is needed. [This could be phrased as multiple choice items, one that asks for additional information and one the asks about assumptions used in modeling.]

Include Problems That Have a Range of Possible Correct Answers

Another problem-solving objective could be one for which various correct answers are possible. Students could be given a set of conditions that need to be fulfilled and asked to generate one of a range of possible answers. A problem scenario might involve the distribution of funds:

> The director of a money-making activity wants to distribute the activity's profits to the 6 student workers. Before the total profit ($24,000) was known, she gave $5,500 to Jeremy, a student who worked on the project. She then discovered the amount of the total profit and a set of requirements for the distribution: (a) she can distribute at most half the profits, (b) amounts given to each student must be in $100 increments, and (c) no two students can receive the same amount. How, if at all, can she follow these directions and distribute the money to the students?

Clarify Different Levels of Complexity

Finally, one of the most promising structures in the current NAEP framework is the level of complexity, a tool that emphasizes the difference among the cognitive demands inherent in different items. This tool is particularly useful in the creation and classification of items that assess problem solving, mathematical modeling, and conceptual understanding. Having served on the Standing Committee on NAEP Mathematics Content, however, I am aware of the difficulty of assigning a level of complexity to particular items. Further clarification of what distinguishes moderate complexity from high complexity would improve the framework.

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