REPORT ON THE 2003 MATHEMATICS SCALE-ANCHORING STUDY

PREPARED UNDER CONTRACT TO AND IN CONJUNCTION WITH THE NATIONAL ASSESSMENT GOVERNING BOARD

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This report has been redacted by the National Assessment Governing Board to remove panelist names (Appendix A)

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Introduction

Scale anchoring is a process that produces descriptions of what students at different levels on a score scale know and can do by examination of student performance on assessment items. Any range on a scale can be "anchored." The principal goal of the 2003 National Assessment of Educational Progress (NAEP) mathematics scale-anchoring study, reported here, was to develop anchor descriptions of the NAEP achievement levels for grade 4 and grade 8 mathematics.

The 2003 National Assessment of Educational Progress (NAEP) mathematics scale-anchoring study was designed to help determine the extent to which scale-anchoring descriptions are affected by the changes in NAEP item pools that occur when items in an assessment are released and replaced, or when assessment frameworks undergo minor revisions. Another goal of the study was to determine how the scale-anchoring descriptions align with the achievement-level descriptions (ALDs) that were developed in the early 1990s. The ALDs describe what students at three levels of mathematics achievement—*Basic, Proficient,* and *Advanced*—should know and be able to do.

NAEP has long used scale-anchoring methodology to describe performance on both the long-term trend and main NAEP assessments. The process involves several steps. First, statistical procedures are used to identify items that "anchor" at given locations on the NAEP scale. Stated simply, to anchor at an achievement level, an item must have a probability of being answered correctly by more than 50 percent of the students at that level (different criteria could be—and have been—used[1]). In addition, the probability of students at the "anchoring level" answering correctly must be substantially higher than the probability for students at lower levels. For example, assume that students performing between scores 299 and 333 at grade 8 (the boundaries of the *Proficient* level) have an 80 percent probability of answering an item correctly. Further, assume that for the same item, students performing at the *Basic* level have only a 45 percent chance of getting the right answer. This question would "anchor" at the *Proficient* level.

After items that anchor are identified statistically, the second step in the process takes place. Panels of subject matter experts are asked to review the items and develop general descriptions of the content knowledge and skills exemplified by the family of items that anchor at a given level. For the present study, the National Assessment Governing Board (NAGB) was interested in

[1] The use of a 50% criterion was used here for alignment with the selection of exemplar items in achievement-level setting studies done for NAEP. In earlier NAEP scale anchoring a criterion of 65% correct probability was used. In the National Adult Literacy Survey, a criterion of 80% correct was employed.

developing descriptions that characterize what students within each of the three achievement levels (*Basic, Proficient,* and *Advanced*), as well as those below the *Basic* level, know and can do. The achievement levels are defined in terms of relatively broad intervals along the NAEP scales. For example, the eighth-grade *Basic* level on the NAEP mathematics scale runs from a score of 262 up to a score of 299, an interval of 37 points. The eighth-grade *Proficient* level runs from 299 up to 333, an interval of 34 points. The eighth-grade *Advanced* level is effectively open-ended, consisting of the range of the scale at or above 333, though few eighth-grade students performed at this level.

To inform the mathematics scale-anchoring study, NAEP convened two panels of mathematics experts (one for grade 4 and a second for grade 8) to review and describe student performance on the 1992 mathematics assessment. Each panel developed item-level descriptors and summary anchor descriptions (ADs) of what students in its grade knew and could do, based on their performance on the assessment. At a meeting eight months later, the same two panels replicated the process for the 2003 mathematics assessment for their respective grades.

In the final step, the NAEP mathematics staff compared the summary anchor descriptions for 1992 and 2003 to each other and each to the mathematics ALDs of what students should know and be able to do in the NAEP mathematics framework, in order to determine how closely the three types of descriptions were aligned. A high degree of overlap between the ADs and the ALDs would suggest a good alignment between the assessment framework that establishes the parameters for developing the test and the two item pools used to construct the 1992 and 2003 mathematics assessments, and between the two item pools. Weaker overlap would suggest some lack of fit, which might result from a number of causes. First, the item pool might not be closely aligned with the content specifications and the ALDs. Second, the item pool might be well aligned with the content specifications, but might be less well aligned with the ALDs (in other words, there may be some mismatch between ALDs and content specifications). Third, students' eventual performance on items may have diverged from what was envisioned by the ALD authors when the ALDs were written in 1990, prior to the development of the 1992 and 2003 assessments. For example, items designed to measure *Proficient* skills may, in fact, have anchored at the *Basic* or *Advanced* levels.

In summary, the scale anchoring process proceeded in three stages. First, statistical analyses were conducted to determine the items that anchored in different achievement level ranges. Second, the two panels of mathematics experts were convened. They reviewed all items that anchored in the different ranges and wrote individual descriptions of the skills and content measured by those items. These panels then created summary descriptions of what students in different achievement-level ranges knew and could do. Third, Educational Testing Service (ETS) mathematics staff completed within-grade-level analyses of the anchor descriptions for the 1992 and 2003 assessments and noted their similarities and differences with the achievement-level descriptions in the *Framework*.

1. Model-Based Approach

The current anchoring study uses a model-based approach,[2] in which individual students are grouped in terms of being in a particular achievement-level interval. After individuals are assigned to an achievement level (based on their NAEP "plausible values"), analysts then compute for each item the probabilities that students in that achievement level will answer individual questions correctly (or, for an open-ended question, reach a given score level).

Historically, ETS has used a "nonparametric" approach to estimating conditional percent correct values (*p*-values). The nonparametric approach has been employed for estimating *p*-values conditional at a particular scale score location, as well as for *p*-values conditional on being within an achievement-level range. In this approach, individuals were grouped in terms of being at or near a scale score or, in the case of interval-level conditional *p*-values, as being within a particular achievement-level interval. The grouping is based on NAEP plausible values of students. Once this grouping is accomplished, ETS simply calculates the *p*-values for the relevant groups of interest. For any given item, the approach uses the data only from those students who were administered that item. In a typical NAEP matrix sample design, this is typically about 20–25 percent of the full sample.

The historical approach makes use of the model-based plausible values that are generated as part of the NAEP analysis for this subset of the sample, but makes no explicit use of the IRT item-parameter estimates that are generated along the way. While this approach has advantages (e.g., that strong assumptions about the functional form of the relationship of item performance to subject-matter proficiency do not play a direct part in the estimates), there is also a downside: restricting the analysis to a portion of those tested often results in inadequate sample sizes to estimate the conditional *p*-values for the *Advanced* level. (This is less of an issue in assessments with combined national-state samples, of course). This problem is particularly evident in NAEP *Report Cards* from years before 2002 and for subjects with national-level samples only, where many of the *Advanced*-level conditional *p*-values cannot be reported because the sample sizes involved fail to meet minimum NAEP standards.

This limitation would have posed intractable problems for this study. In order to develop anchor descriptions of all of the NAEP achievement levels in a subject in a year with a small sample available (in this case, mathematics from

[2] The model-based approach is described in detail in appendix C of Stephen Lazer, John Mazzeo, and Andrew Weiss, Final Report on Enhanced Achievement-Level Reporting and Scale-Anchoring Activities (2000).

1992), we needed to produce reasonable estimates of conditional *p*-values at all achievement levels, not just those where sample sizes supported the existing approach. To do this, ETS adopted a "model-based" approach to estimating conditional *p*-values (an approach that was previously used in the Geography scale-anchoring studies conducted for NAGB). The model-based approach uses the full NAEP sample, and hence avoids the sample-size problems encountered with the historical ETS approach. It does so, however, by relying more explicitly on the assumptions of the IRT models used in NAEP and makes explicit use of the IRT item-parameter estimates used in NAEP.

This approach is also more similar to the approach ACT has used in selecting potential exemplar exercises as part of the achievement-level-setting studies. However, we have adopted a different calculation method from ACT.

NAGB instructed ETS to use the same statistical criteria for this study that ACT used in its exemplar-selection work. ACT has traditionally used a 50 percent conditional *p*-value standard in its NAEP exemplar selection work. Items qualify as anchoring at an achievement level if the conditional *p*-value equals or exceeds 50 percent. When items meet this criterion for more than one achievement level, the item is classified as a potential exemplar for the lowest level at which the criterion is met. ACT has also imposed a discrimination criterion. In the ACT procedures, the item discrimination at a particular level is defined as the difference between the item's conditional *p*-value for that level and its conditional *p*-value at the next lower level. An item therefore has three discrimination values (*Basic* minus below *Basic, Proficient* minus *Basic,* and *Advanced* minus *Proficient*). To meet the ACT discrimination criterion at a particular level, an item must have a discrimination value at that level that is at or above the 40th percentile of the distribution (across items) of discrimination values at that level.

Using these processes and criteria, ETS Research staff analyzed all items from the 1992 and 2003 NAEP mathematics assessments and determined which items mapped into given achievement-level ranges. Tables 1 and 2 show the number of items (or score points on open-ended questions) that anchored in each range at each grade. Based on their conditional *p*-value, items might be placed into one of six row categories, beginning with those items that anchored below the *Basic* level.

In addition to anchoring in one of the four regions defined by the three achievement-level cut scores, items might be statistically classified in two other ways: "did not discriminate between levels," and "did not anchor" (last two rows of tables 1 and 2). Some items did meet the first criterion (i.e., their conditional *p*-value was greater than 50 percent) for anchoring in a range, but did not discriminate adequately with lower levels. Finally, a small number of difficult items did not anchor at any level, because students at no achievement level had a 50 percent likelihood of answering correctly (or reaching a given score level).

One general caveat should be offered about the data in tables 1 and 2. We often discuss whether or not "items" anchor in a given range. This is an apt depiction of any item (such as a multiple-choice question) that is scored right or wrong (i.e., a dichotomously scored item). However, items with partial credit scoring may anchor in several places. For example, for an open-ended item scored with a four-point scoring guide (scored as 1, 2, 3, or 4), there are three possible dichotomizations, score 1 vs. score 2 and above, score 2 and below vs. score 3 and above, and score 3 and below vs. score 4. In other words, an item with a four-point guide will appear to be three (dichotomous) items in the anchoring process analysis. Clearly, these three-score-level items have quite different difficulty levels. Therefore, it is quite possible that, for example, the low-score-level response to an item anchors below the *Basic* level, the middle-score-level response at the *Proficient* level, and the high-score-level response at the *Advanced* level. Similarly, an item with a three-point guide will appear to be two (dichotomous) items in the anchoring process analysis. For this reason, the total number of items (called items/score levels) in any of the columns in tables 1 and 2 is greater than the number of discrete items on the assessment.

Table 1. Numbers and percentages of NAEP mathematics items, anchoring across categories, grade 4: 1992 and 2003

	1992 Assessment		2003 Assessment	
	Number of Items/Score Levels	Percentage of all Items/Score Levels	Number of Items/Score Levels	Percentage of all Items/Score Levels
Below the *Basic* level	25	15	37	18
Basic	51	31	56	27
Proficient	51	31	62	29
Advanced	26	15	37	18
Did not discriminate	6	4	12	6
Did not anchor	8	5	6	3

NOTE: Because responses to some items were scored at multiple levels, column totals may be greater than the number of items in the assessment. Detail may not sum to totals because of rounding.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1992 and 2003 Mathematics Assessments, 2003 Mathematics Scale-Anchoring Study.

Table 2. Numbers and percentages of NAEP mathematics items, anchoring across categories, grade 8: 1992 and 2003

	1992 Assessment		2003 Assessment	
	Number of Items/Score Levels	Percentage of all Items/Score Levels	Number of Items/Score Levels	Percentage of all Items/Score Levels
Below the *Basic* level	55	28	55	25
Basic	62	32	60	27
Proficient	41	21	54	24
Advanced	25	13	35	16
Did not discriminate	2	1	9	4
Did not anchor	10	5	9	4

NOTE: Because responses to some items were scored at multiple levels, column totals may be greater than the number of items in the assessment. Detail may not sum to totals because of rounding.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1992 and 2003 Mathematics Assessments, 2003 Mathematics Scale-Anchoring Study.

2. Review of Items and Score Points by Anchor Panel

The Anchor Panel

The next step in the process was the convening of a grade 4 panel of mathematics experts and a grade 8 panel of mathematics experts to review the results of the anchoring and to produce written descriptions of the content knowledge and skills displayed by students within each achievement-level range. Each panel included at least one university-level mathematics faculty member, at least one state mathematics specialist, and at least one mathematics classroom teacher at the appropriate grade level. A list of the panelists and other attendees of the anchoring meeting can be found in appendix A.

Anchor Panel Activities

The two meetings of the panel began with an orientation session. Sharif Shakrani, from the NAGB staff, discussed the goals of the scale-anchoring study. He explained the potential usefulness of anchoring for studying the alignment between the achievement levels and the assessment instrument, for evaluating the impact of changes in the item pool over time on the reporting of assessment results, and for enhancing future NAEP analyses. Andy Weiss, from ETS, described the anchoring process, the statistical analyses that produced the anchoring data, and the procedures the panels would follow in their work of describing the assessment content.

Following the orientation session, the panelists began writing individual item descriptors and summary descriptions of what students know and can do in each of the three achievement-level ranges (*Basic*, *Proficient*, and *Advanced*) and below the *Basic* level. First, panelists reviewed an item, its associated anchoring data, and, in the case of constructed-response questions, the scoring guide. Then, after some discussion, they wrote a description of the knowledge and skills demonstrated by students who answered the question correctly. In the case of constructed-response questions, the descriptors referred to the knowledge and skills demonstrated by students receiving the particular score—for example, “partial” or “complete”—that anchored in the achievement-level range being reviewed. Generally, different levels of performance on constructed-response questions anchored at different achievement levels, but when more than one score point anchored at the same level, the panelists would describe the knowledge and skills associated with the higher score.

At the first meeting, the panel began with the Number Sense, Properties, and Operations content area. Starting with items that anchored below the *Basic* level, the panel worked their way up through those anchoring at *Basic*, *Proficient*, and *Advanced*. Within each achievement-level range, the panels moved from the easiest items (highest conditional *p*-value) to the most difficult items (lowest conditional *p*-value). In this way, the panelists could see a progression in what students knew and were able to do as they worked through the item pool. In other words, panelists would first see items that were likely to be answered correctly by most of the students in an achievement-level range, and would progress to those items that had been answered correctly by fewer students. Items answered correctly by fewer than half the students in a range would not be judged to anchor in that range. The panels then proceeded to work through the items as described above for each of the four remaining content areas (Measurement; Geometry and Spatial Sense; Data Analysis, Statistics, and Probability; and Algebra and Functions).

Before the first meeting, NAEP staff organized the item pools for each grade into notebooks based on experiences with the geography anchoring study. For that meeting, each notebook was divided into six sections, one for each of the categories of the anchoring analysis:

- Below the *Basic* level
- *Basic*
- *Proficient*
- *Advanced*
- Items that did not anchor because they were too difficult (fewer than 50 percent of students at the *Advanced* level answered correctly)
- Items that did not anchor because they did not meet the discrimination criteria

Although items in the notebooks for the first meeting were organized by achievement-level ranges and by content within each range, the panels did not review them in this order. As mentioned above, the panels considered each content area in turn, starting with items that anchored below the Basic level and moving up to higher achievement levels. Since this order of review worked well for the first meeting, the organization of the notebooks for the second meeting was changed to reflect this order of review.

For the second meeting, each notebook was divided into the following seven sections:

- Number Sense, Properties, and Operations
- Measurement
- Geometry and Spatial Sense
- Data Analysis, Statistics, and Probability
- Algebra and Functions
- Items that did not anchor because they were too difficult (fewer than 50 percent of students at the *Advanced* level answered correctly)
- Items that did not anchor because they did not meet the discrimination criterion

Within each content area items were arranged from those that anchored below the *Basic* level to those that anchored at *Basic*, and so forth, through those that anchored at *Advanced*.

After writing descriptors for each item, the panelists distilled and summarized what students performing in that content area and range knew and could do. To accomplish this task, they reviewed the item descriptors, grouping together those that described similar skills or content knowledge. Depending on the weight of the "evidence," the panelists could then make statements with varying degrees of certainty. For example, if a number of questions demonstrating an understanding of multiplication of fractions anchored at a particular level, the panelists could state with some confidence that students could multiply fractions. If, on the other hand, students had answered only one or two questions on a topic, or had demonstrated "partial" mastery on a constructed-response question, then panelists would be likely either to omit the topic or to use modifying language ("some knowledge," "beginning understanding") when describing what students know and can do. The panels wrote descriptors and first drafts of summary descriptions for each content area and range before moving on to the next content area and range. At a later time, NAEP staff reorganized the summary descriptions to make comparisons across descriptions easier, submitted the revised descriptions to the panels for review, and incorporated their final comments.

The summary anchor descriptions developed by the two panels for the 2003 assessment appear in appendix B. They are displayed alongside the achievement-level descriptions and the summary anchor descriptions for the 1992 assessment.

3. Results of the 2003 Scale-Anchoring Study

As discussed in section 1, this study was designed to permit comparisons between the 1992 and 2003 mathematics assessments as well as between the assessments and the ALDs, with the purpose of discerning significant shifts in content over time and drift from the achievement-level descriptions. By having the same panels develop the ADs for two different assessments, the study hoped to minimize differences that might arise from having different groups of experts evaluating the item pools. Differences in the ADs that did appear would more likely reflect actual differences in the item pools. The analyses that follow are based on the summary anchor descriptions of the 1992 and 2003 assessments produced in February 2003 and October 2003, respectively.

Comparison of Anchor Descriptions of the 1992 and 2003 Assessments for Grade 4

Below the *Basic* Level Comparisons

For the **Number Sense, Properties, and Operations** area, analysis of performance on items that anchored below the *Basic* level in both 1992 and 2003 indicated that students performing below the *Basic* level were able to implement simple routine procedures presented in a familiar way. In the area of computation, in both years they exhibited a better understanding of addition and subtraction than of multiplication and division. In 1992, students below the *Basic* level were able to add and subtract two- or three-digit whole numbers that required a single regrouping, while in 2003, they were able to perform addition and subtraction with small whole numbers. In both years, students were able to recognize common pictorial representations for fractions.

In the **Measurement** area, analysis of performance on items that anchored below the *Basic* level in both 1992 and 2003 showed that students performing below the *Basic* level could perform very rudimentary tasks with common measurement instruments and measurement units.

In the **Geometry and Spatial Sense** area, analysis of performance on items that anchored below the *Basic* level in both 1992 and 2003 showed that students performing below the *Basic* level exhibited very similar skills. In both years, students could recognize simple geometric shapes.

In the area of **Data Analysis, Statistics, and Probability** in 2003, students below the *Basic* level could read simple pictographs and bar graphs. No items in this area anchored below the *Basic* level in 1992.

In the **Algebra and Functions** area, no items anchored below the *Basic* level in either 1992 or 2003.

Generally, there was little evidence in either year that students could apply mathematics in contextual situations.

Basic Level Comparisons

For the **Number Sense, Properties, and Operations** area, performance on items that anchored at the *Basic* level in both 1992 and 2003 showed that students performing at this level could complete addition and subtraction problems involving multiple regroupings, as well as basic multiplication problems. They could also answer questions involving place value.

In the **Measurement** area, analysis of performance on items that anchored at the *Basic* level in both 1992 and 2003 showed that students performing at this level could read common measurement instruments.

In the **Geometry and Spatial Sense** area, analysis of performance on items that anchored at the *Basic* level showed that students performing at the *Basic* level achieved a moderate level of success with problems involving the manipulation of geometric shapes. In 1992, students at this level demonstrated an increased growth beyond students who were below the *Basic* level in working with comparative phrases and geometric terminology. In 2003, they were beginning to visualize familiar shapes in two and three dimensions. In both years students at this level possessed a beginning, rather than a firm, knowledge of the concepts and acquisition of the skills expected of fourth-grade students.

In the area of **Data Analysis, Statistics, and Probability**, analysis of performance on items that anchored at the *Basic* level indicated that in both 1992 and 2003 students could complete simple problems involving reading graphs and tables and were developing a beginning understanding of probability.

In the **Algebra and Functions** area in 1992, students performing at the *Basic* level had a beginning, rather than a firm, knowledge of the concepts and skills expected at fourth grade. In 2003, students at this level showed some ability to use comparative reasoning and their problem solving abilities to solve one-step problems. In 2003, students performing at the *Basic* level could work with simple patterns. They were beginning to demonstrate an ability to use mathematics in questions that are less structured (i.e., where the procedure to be implemented is not as obvious). The differences between students at the *Basic* level in 1992 and students at the *Basic* level in 2003 centered on a small number of questions involving graphs and patterns.

***Proficient* Level Comparisons**

For the **Number Sense, Properties, and Operations** area, analysis of performance on items that anchored at the *Proficient* level in 1992 and 2003 indicated that students performing at this level were able to complete problems involving operations with whole numbers, solve problems using division (an operation not typically well understood by grade 4 students at the *Basic* level), often in different content areas of mathematics, and interpret the meaning of remainders in context. Students performing at the *Proficient* level in 1992 could use simple fractions to measure lengths. In 2003, students at this level were able to use fractions to determine the number of unit fractions in a whole, use region models for fractions, and locate fractions on a number line.

In the **Measurement** area, analysis of performance on items that anchored at the *Proficient* level in 1992 showed that students performing at this level could use reasoning in solving two-step problems that required decisions in the solution process. In 2003, they could solve similar types of problems. For example, they could solve an elapsed-time problem involving half-hour increments in which the change from a.m. to p.m. needed to be taken into account in the solution process.

In the **Geometry and Spatial Sense** area, analysis of performance on items that anchored at the *Proficient* level showed that in 1992 students performing at this level could produce geometric shapes that were described by multiple conditions and could use their knowledge of simple figures in less familiar situations. In 2003, students were using geometric visualization and reasoning in solving two-step problems that required decisions in the solution process. In both years, students seemed to be moving beyond mere recognition of geometric shapes to using simple geometric concepts in problem situations.

In the area of **Data Analysis, Statistics, and Probability**, analysis of performance on items that anchored at the *Proficient* level showed that students in both years could create and interpret data in various types of graphs, including bar graphs and pie charts, and could identify simple probabilities.

In the **Algebra and Functions** area, analysis of performance on items that anchored at the *Proficient* level showed that in 1992, students performing at the *Proficient* level had greater use and understanding of grade 4 mathematics and in more complex situations than students at the *Basic* level. They could interpret and solve problems that required two steps, use information from more than one source, show flexibility in their thinking, and could use a wide spectrum of appropriate grade-4 mathematical concepts and skills to complete tasks that were not always inherently obvious. In 2003, students at this level demonstrated increased understanding and use of patterns in unfamiliar situations and could use reasoning in solving problems that required decisions in the solution process. As in 1992, in 2003, students who were at the *Proficient* level seemed to have a greater understanding of grade 4 mathematics than students at the *Basic* level.

Advanced Level Comparisons

At this level, examining the content areas separately showed less in the way of notable differences. Therefore, this comparison is more holistic than those presented previously.

Students performing at the *Advanced* level in 1992 could solve a wide variety of problems and could explain their solutions using words and pictures. They could interpret mathematical concepts and language in complex and novel situations, and were able to think flexibly to solve problems with more than two conditions or steps. Students performing at the *Advanced* level in 2003 could also solve multi-step problems and were able to process multiple conditions while solving the problems. In both years there was a greater understanding of fractions demonstrated than by students who were at the *Proficient* level in those years. Students at the *Advanced* level were able to communicate better than students at the lower levels, interpret mathematics in more novel situations, and showed growth in the use of their mathematical language.

Summary of Comparisons of Fourth-Grade Anchor Descriptions, 1992 and 2003

While there were minor differences between what fourth-grade students could do at each level in 1992 compared with what they could do at the corresponding level in 2003, the similarities between the years at each level were strong. Below the *Basic* level, students in both years could compute with addition and subtraction, but were unable to work with mathematics in any but the simplest situation. At the *Basic* level in both 1992 and 2003, students added facility with multiplication to their computational repertoire and were beginning to apply mathematics in different contexts in most of the content areas. There was some evidence in 1992, but not in 2003, that students could read simple graphs, and there was some evidence in 2003, but not in 1992, that students could work with simple patterns. In both years, students at the *Proficient* level demonstrated an understanding of division, as well as the ability to use mathematics in two-step problems in a wide variety of contexts. They were able to begin to use more complex thought processes in solving problems. At the *Advanced* level, students in both years were routinely able to complete complex exercises.

Comparison of Anchor Descriptions of the 1992 and 2003 Assessments for Grade 8

Below the *Basic* Level Comparisons

For the **Number Sense, Properties, and Operations** area, analysis of performance on items that anchored below the *Basic* level in both 1992 and 2003 showed that eighth-grade students performing below the *Basic* level could perform simple computations involving addition, subtraction, and long division without the use of a calculator. In both years, they could recognize fractions represented by shaded regions and identify decimals in tenths represented on a number line, and they demonstrated some understanding of place value, rounding, and integer multiples. In 1992, students were also able to identify the representation of equivalent fractions in a region model. In both years, students were able to solve a variety of one-step and very simple two-step word problems involving various arithmetic operations, some involving money. In both years, some of these problems were in blocks that permitted calculator use. The variety of word problems students were able to solve at this level was greater in 1992 than in 2003. For example, in 1992, there were 11 word problems that anchored below the *Basic* level, but only 5 such problems anchored below the *Basic* level in 2003. The word problems in 1992 involved the application of a greater variety of arithmetic operations as well as the use of estimation skills that were not so evident in 2003.

In the **Measurement** area, analysis of performance on items that anchored below the *Basic* level in both 1992 and 2003 showed that students could solve simple weight problems, read weights from a scale, convert from one customary unit to another, find the area of an irregular figure on a rectangular grid, and estimate the reasonableness of measures for given situations. In 1992, students were also able to measure length to the nearest centimeter, identify the instrument used to find angle measure, and compare areas of simple figures. In 2003, students were able to order given angles, and solve simple capacity and length problems.

In the **Geometry and Spatial Sense** area, analysis of performance on items that anchored below the *Basic* level in both 1992 and 2003 showed that students exhibited very similar skills. In both years, students could recognize examples and properties of common geometric figures, identify the result of paper folding, recognize simple transformations of figures, visualize the result of cutting and unfolding a figure, and identify a counterexample to a statement about a rectangle. In 1992, students were able to use geometric shapes to form simple figures and to explain how certain cardboard shapes differed.

In the area of **Data Analysis, Statistics, and Probability**, analysis of performance on items that anchored below the *Basic* level was very similar in both years. In both 1992 and 2003, students could read, interpret, and draw simple inferences from bar graphs, tables, and pie charts. They could also use given information to complete a simple bar graph or pictograph.

In the **Algebra and Functions** area, analysis of performance on items that anchored below the *Basic* level in both years showed that students could identify missing terms in a simple visual pattern, evaluate numerical expressions using order of operations, and find a missing value in a simple number sentence. In 1992 students could identify a simple expression involving a variable that represented a problem situation. In 2003, students could also solve simple one- and two-step equations involving small whole numbers, solve a simple word problem involving pairs of whole numbers, extend a number pattern, and solve a simple problem involving coordinates on a map.

Basic Level Comparisons

For the **Number Sense, Properties, and Operations** area, analysis of performance on items that anchored at the *Basic* level in both 1992 and 2003 showed that students could solve problems involving equivalent fractions and could represent fractions using both region and number line models. They could also apply place value concepts to solve problems or to round numbers. They could solve one-step word problems involving a variety of operations, but had limited success in solving two-step problems. However, in 1992, students could solve a multi-step word problem involving money. In 2003, students could identify extraneous information in a word problem, solve simple percent problems, compare rates in context, and identify equivalent ratios.

In the **Measurement** area, analysis of performance on items that anchored at the *Basic* level in both 1992 and 2003 showed that students could determine lengths in customary units and could solve problems that required the reading of information on scales or other measurement devices such as dials. In 1992, students could also determine length in metric units and solve problems involving the perimeter and area of rectangles. In 2003, students could locate mixed numbers and fractions on a number scale, draw angles greater than or smaller than a right angle, and find the measure of a missing angle in a triangle.

In the **Geometry and Spatial Sense** area, analysis of performance on items that anchored at the *Basic* level showed considerable difference between the two years. In 1992, students could draw simple geometric figures given one or more conditions, solve problems involving two-dimensional figures that could be folded to form a cube, compare the measure of an angle to a right angle, and identify the reflection of a point located on a geometric figure. In 2003, students were able to partition shapes with or without the use of manipulatives, demonstrate some understanding of reflections, rotations, and symmetry with or without the use of manipulatives, use spatial visualization with manipulatives to identify the possible shape when two common figures overlap, and solve a standard problem involving side lengths of similar triangles. So the exemplars at the *Basic* level in this content area for the two years were very different.

In the area of **Data Analysis, Statistics, and Probability**, analysis of performance on items that anchored at the *Basic* level showed that in both 1992 and 2003 students could solve simple problems involving probability, draw conclusions about displayed data (tables, bar graphs, stem-and-leaf plots), and evaluate simple issues related to surveys such as design and sample size. In 2003, students were also able to find the median of a set of values.

In the **Algebra and Functions** area, analysis of performance on items that anchored at the *Basic* level in both years showed that students performing at this level could extend patterns and demonstrate evidence of understanding the rule used to generate the pattern. They could solve a variety of equations such as linear, pictorial, or a simple equation involving a square root. In both years, the problem solving of eighth-grade students performing at *Basic* demonstrated a higher level of understanding and skill than students below the *Basic* level.

Proficient Level Comparisons

For the **Number Sense, Properties, and Operations** area, analysis of performance on items that anchored at the *Proficient* level in both 1992 and 2003 showed that students performing at this level could solve one- and two-step word problems of greater complexity than students at the *Basic* level and below. For example, in both years, students could solve word problems involving percentages and proportional reasoning. They were also able to use reasoning and problem-solving skills to generate examples or counterexamples about number relationships, and they could explain the reasoning behind their answers. In 1992, they could order fractions with simple, unlike denominators. In 2003, they could identify a number to the thousandths place on a number line.

In the **Measurement** area, analysis of performance on items that anchored at the *Proficient* level in both 1992 and 2003 showed that students could use manipulatives to analyze and compare the areas of shapes. In both years, they could solve rather complex problems involving the perimeter of a polygon or the area of a rectangle. In 1992, students could use a protractor to find the degree measure of an angle. In 2003, they could find the measure of a missing acute angle in a right triangle.

In the **Geometry and Spatial Sense** area, analysis of performance on items that anchored at the *Proficient* level showed considerable difference between the two years. This was also true at the *Basic* level. In 1992**,** students performing at the *Proficient* level could solve problems involving interior and exterior angle measures in triangles. They could identify correct and incorrect statements about the properties of common geometric figures such as circles, parallelograms, or triangles. They could create a drawing to fit a verbal description of points on a line and also could draw a line of symmetry for geometric figures. In 2003, students could use spatial visualization to identify the result of folding paper, could identify the reflection of a point in a nonstandard context, could recognize properties of a given quadrilateral, and could identify the proper name for a special type of triangle. Students could also apply the Pythagorean theorem.

In the area of **Data Analysis, Statistics, and Probability**, analysis of performance on items that anchored at the *Proficient* level showed that in both 1992 and 2003 students could solve problems using data read from a graph and could solve probability problems of greater complexity than students performing at the *Basic* level. In 1992, students could also identify sets of data that have a given average and apply percents given in a pie chart. In 2003, students could recognize bias in a sample and find the median of a set of values that was even in number.

In the **Algebra and Functions** area, analysis of performance on items that anchored at the *Proficient* level in both years showed that students could analyze and extend nonroutine patterns, solve problems involving two variables, apply a common formula, graph inequalities on a number line, and solve problems involving points in the coordinate plane. In 2003, students were also able to identify possible solutions to a multi-step logic problem, solve an equation for one variable in terms of another, and solve a problem involving the *x*-intercept of a function.

Advanced Level Comparisons

For the **Number Sense, Properties, and Operations** area, analysis of performance on items that anchored at the *Advanced* level in both 1992 and 2003 showed that students performing at this level were only partially successful in solving extended constructed-response questions. There was only one such extended question in this content area each year, and the question in 1992 was different from the question in 2003. Both questions were fairly complex and required the student to provide an explanation. The percentage of students who achieved the upper level scores was not at the 50% level needed to anchor at the *Advanced* level. In both years, students were able to answer a question about properties of odd numbers. In 1992, students could also interpret a number pictured on a calculator screen with an implied exponent. In 2003, students could solve a multi-step problem involving a tip on a restaurant bill and sharing the total charge. For each year, there were very few items (3 in 1992 and 4 in 2003) that anchored at the *Advanced* level in this content area.

In the **Measurement** area, analysis of performance on items that anchored at the *Advanced* level in both 1992 and 2003 showed that students performing at this level could solve problems related to accuracy of measurement, and could determine area in a variety of contexts, such as finding the surface area of a solid, partitioning an irregular figure to find the areas of its component parts, or tiling a rectangular region. In some of these problems, students had to explain their reasoning. In 1992, students could also find the perimeter of a figure using a nonstandard unit. In 2003, students could use a protractor to draw a directional angle and could solve a nonroutine problem involving arc length and estimation.

In the area of **Geometry and Spatial Sense**, analysis of performance on items that anchored at the *Advanced* level showed that students could use given information about an unspecified quadrilateral to draw an additional conclusion about the figure's properties. They could also solve problems relating central angles of circles to arc length. In 1992, students could also apply the Pythagorean theorem, recognize a construction that would produce a familiar angle, and identify the intersection of two common geometric figures. In 2003, students could use manipulatives to create complex shapes with specified properties, solve a problem related to placing spherical objects in a cylindrical container, use interior and exterior angle relationships in a triangle to find a missing angle, and demonstrate an understanding of the relationship among classes of quadrilaterals.

In the area of **Data Analysis, Statistics, and Probability**, analysis of performance on items that anchored at the *Advanced* level showed that in both 1992 and 2003 students could find the average for numbers given in a frequency distribution table, find the probability of an event involving ordered pairs of numbers, and list the sample space for a situation involving sampling with replacement. In 1992, students could determine the median of a set of numbers either from a list or a scatter plot. They could also use the angle measure of a sector of a circle to solve a problem. In 2003, students could evaluate statements about data presented in a pie graph and also solve multi-step word problems such as reading data from one bar graph and using it in conjunction with data from another bar graph.

In the **Algebra and Functions** area, analysis of performance on items that anchored at the *Advanced* level in both years showed that students could analyze and explain complex patterns presented in numerical or geometric contexts. In 1992, students could evaluate a numerical expression involving a variety of operations, using order of operations. In 2003, students could solve word problems involving two variables, approximate the solution to a word problem by extending two lines to their point of intersection, identify the graph of a linear function, and relate the magnitude of a change in one variable in a linear equation to the corresponding change in the other variable. Some of these problems required explanations. The algebra questions that anchored at the *Advanced* level were more numerous and varied in 2003 than in 1992.

Summary of Comparisons of Eighth-Grade Anchor Descriptions, 1992 and 2003

It is clear from the comparisons above that there was considerable similarity between what eighth-grade students knew and could do at each level in 1992 and 2003. There were a few differences worth noting. In 1992, students performing below the *Basic* level were able to solve a wider range of problems in the **Number Sense, Properties, and Operations** area than students in 2003. However, in 2003, students performing below the *Basic* level were able to solve a richer variety of simple **Algebra** problems than in 1992. Performance was similar in other content areas at this level. At the *Basic* and *Proficient* levels, the types of **Geometry** problems students could solve were quite different, but in each case the demand level was reasonably consistent with the achievement level. These differences probably relate to the nature of the item pools used in the two assessment years. Since items in the assessments are regularly released and replaced by new items, it is reasonable to expect some differences in the descriptions for items that anchored in each achievement-level range between 1992 and 2003.

Comparison of the Anchor Summary Descriptions to the Achievement-Level Descriptions

The final section of this report addresses NAGB's interest in how well the summary anchor descriptions (ADs) align with the achievement-level descriptions (ALDs) and whether there is drift in that alignment over time as assessment items change. This information must be considered if NAGB moves towards enhancing the results NAEP reports by including scale-anchoring descriptions.

It is useful first to consider the differences between the ALDs and the ADs. The NAEP achievement levels are a set of performance criteria that outline what students should know and be able to do to meet certain standards. As is the case with all NAEP subjects, the achievement levels for mathematics are based upon a set of achievement-level descriptions (developed as part of the national consensus process to determine the assessment design and content) and a set of achievement level cut points on the NAEP scale. Alignment should occur naturally between the ALDs and the assessments, at least to an extent, because the assessments were developed with the ALDs in mind. Moreover, the ALDs and assessments should be aligned, because the ALDs were written as part of the mathematics framework development process, and the assessments were written to meet content specifications set forth in the framework.

By comparing the ALDs to the ADs developed for the anchoring study, one gains some information about the extent of alignment between the ALDs as standards and the actual knowledge and skills demonstrated by students on the 1992 and 2003 assessments. One also gains information about whether alignment has strengthened or weakened over time. Since both the writing of the descriptions and the comparisons among them are subjective processes, there is certainly no reason to expect precise alignment between them, and the results of the process should be treated as suggestive rather than definitive. Inherent sources of imprecision should be kept in mind. First, different groups of individuals wrote the different sets of descriptions. Second, the ALDs were written mostly in the abstract whereas the ADs were written based directly upon the assessment instruments. Third, the very language educators use to describe knowledge and performance evolved somewhat between the early 1990s, when the ALDs were written, and 2003, when the ADs were written. For all these reasons, one would expect to find differences in the language used to create the descriptions.

Comparisons between the ALDs and the 1992 and 2003 mathematics assessment ADs are based on reviews conducted by ETS staff members who developed and oversaw the mathematics assessment.

Because there are no achievement-level descriptions for performance below the *Basic* level, there can be no explicit comparison made for the below-*Basic* summary anchor descriptions. Comparisons of the *Basic*-level ALDs must be made to a combination of the *Basic* and below-*Basic* ADs, since it is assumed that students at the *Basic* level have mastered the skills and knowledge described for below *Basic*. For example, the *Basic* ALD prescribes that students should be able to show some understanding of fractions. Mention of this skill is absent from the *Basic* AD, but is included in the below *Basic* AD. One can conclude, therefore, that on this topic the *Basic* ALD and AD are aligned.

Comparison of Achievement-Level Descriptions for Grade 4 to the 1992 and 2003 Anchor Descriptions for Grade 4

Basic **Level Analysis**

The *Basic* ALD states that students performing at *Basic* show some understanding in all five of the content areas. In 1992, there was evidence of understanding in each content area except Algebra. However, below the *Basic* level, there was evidence that students could work with patterns, which is part of the Algebra content area. In 2003, there was evidence of understanding in each content area except Data. However, below the *Basic* level, there was evidence that students could read simple pictographs and bar graphs, which is part of the Data content area. The *Basic* ALD further states that students can estimate and use basic facts to perform simple computations with whole numbers, show some understanding of fractions and decimals, and solve simple real-world problems. In 1992, students could perform computations (especially involving addition, subtraction, and multiplication) and solve simple real-world problems. Below the *Basic* level, they were able to work with fractions in rudimentary problems. In 2003, students could perform computations (especially involving addition, subtraction, and multiplication) and possessed some understanding of place value. Below the *Basic* level, they were able to work with common pictorial representations for fractions. Finally, the *Basic* ALD states that students can use four-function calculators, rulers, and geometric shapes with some success, and that at this level their written responses are often minimal and presented without supporting information. In 1992, students were able to read common measurement instruments and attain moderate success with the manipulation of geometric shapes. While they were beginning to work with some mathematical vocabulary, they were unable to formulate written justifications. In 2003, students were able to read common measurement instruments and attain moderate success with the manipulation of geometric shapes. They showed some ability to employ comparative reasoning and problem-solving abilities to solve one-step problems.

Proficient Level Analysis

The ALD states that students performing at the *Proficient* level can consistently apply integrated procedural knowledge and conceptual understanding to problem solving in the five content areas, can use whole numbers in computations and problem situations, have a conceptual understanding of fractions and decimals, can solve real-world problems, and use calculators, rulers, and geometric shapes appropriately. The 1992 AD for students performing at *Proficient* states that students can solve problems involving whole numbers, have acquired an understanding of division, can interpret remainders from division in context, use fractions in various contexts, and can interpret and solve problems that require two steps or use information from more than one source. They also show flexibility in their thinking and can use grade-appropriate concepts and skills to solve tasks that are not always inherently obvious. The 2003 *Proficient* AD states that students can solve problems involving whole numbers, work with division in different contexts, use fractions in various situations, and use reasoning in solving problems that require decisions in the solution process.

Advanced Level Analysis

At the *Advanced* level, the ALD states that students can apply integrated procedural knowledge and conceptual understanding to complex and nonroutine real-world problems in all content areas. The students are expected to draw logical conclusions and justify answers and solution processes by explaining why, as well as how, the conclusions and answers were achieved. They should go beyond the obvious in their interpretations and be able to communicate their thoughts clearly and concisely. The 1992 *Advanced* AD indicates that students can solve a wide variety of problems and can explain their solutions using pictures and words, as well as interpret mathematical concepts and language in complex and novel situations. These students are also able to reason and think in different ways to solve problems with more than two conditions or steps. The 2003 *Advanced* AD indicates that students can solve multi-step problems and can process multiple conditions while solving those problems. They can explain their solutions clearly and have demonstrated substantial growth in the use of mathematical language.

Overall, the alignment between the ALDs and the 1992 and 2003 ADs is very strong for the grade 4 assessment. At the *Basic* level, the ALD and the 1992 and 2003 ADs indicate that students possess some rudimentary understanding in a variety of content areas, but generally are unable to employ reasoning in the solution of problems. At the *Proficient* level, their reasoning skills are beginning to be developed. And, at the *Advanced* level, students can reason and think in different ways, demonstrate solid command of mathematics, and are able to communicate their understanding.

Comparison of Achievement-Level Descriptions for Grade 8 to the 1992 and 2003 Anchor Descriptions for Grade 8

Basic Level Analysis

The *Basic* ALD states that eighth-graders are expected to show evidence of understanding in all five content areas. An examination of the *Basic*-level summary descriptions for 1992 and 2003 in Appendix B, together with the grade 8 ADs described earlier, shows that students exhibit ample evidence of understanding in all five content areas. This is especially evident when considered in conjunction with what students performing below the *Basic* level could do.

Students at the *Basic* level are expected to be able to show understanding of operations on whole numbers, fractions, and percents. With the exception of percents in 1992, each of these operations was well represented in both 1992 and 2003. For example, in both years, students could represent fractions using either number line or region models.

Students at the *Basic* level are expected to be able to use structural prompts such as diagrams, charts, and graphs. Of the items that anchored below the *Basic* level in both 1992 and 2003, several involved reading and/or interpreting information presented in diagrams, graphs, or charts. Items that anchored at this level required students to draw conclusions about data displayed in various ways (1992) or to evaluate data displays (2003).

Students at the *Basic* level are expected to be able to solve problems in all NAEP content areas. This was clearly true in both 1992 and 2003 in the Number Sense, Properties, and Operations content area. In both years, students below and at the *Basic* level could solve one-step problems involving various types of numbers and operations. Students at this level had limited success in solving multi-step problems in both years. In 1992, students could solve measurement problems involving the perimeter and area of rectangles, geometry problems involving folding figures and reflecting points about a line, simple probability problems, and algebra problems involving patterns and variables. In 2003, students could solve measurement problems involving weight or time, geometry problems involving spatial visualization, simple probability problems, and algebra problems involving patterns. So, students performing at the *Basic* level in both 1992 and 2003 could solve a variety of problems in all five content areas. Across the two years, some of the content of the ADs was similar (such as probability and algebraic patterns) and other content was different, probably because some questions were unique to one of the assessment years.

The *Basic* ALD states that as students approach the *Proficient* level they are able to determine which of the available data in a problem situation are necessary and sufficient for solving the problem. Only in 2003 was there some evidence of success in this area. This evidence relates to students being able to identify extraneous information in a word problem. It was observed that students at the *Basic* level had very limited success in solving extended constructed-response questions and short constructed-response questions requiring an explanation. The generally poor performance of eighth-graders performing at the *Basic* level on such questions in both 1992 and 2003 is consistent with the ALD for the *Basic* level, which states that students at the *Basic* level have limited skill in communicating mathematically.

***Proficient* Level Analysis**

The *Proficient* ALD states that eighth-graders performing at this level are able to conjecture, defend their ideas, and give supporting examples. There was substantial evidence in both 1992 and 2003 ADs that students could engage in these behaviors. For example, in both years students could generate examples or counterexamples about number relationships and explain the reasoning behind their answers. Also, they were able to analyze and extend nonroutine patterns, a skill that requires some conjecturing about the nature of the rule for generating the pattern.

The *Proficient* ALD also states that students should understand the connections among topics and have an understanding of *Basic*-level arithmetic sufficient for solving problems in practical situations. In both 1992 and 2003, students were able to solve word problems involving percents and proportional reasoning. They were also able to apply computation skills in other areas, such as reading data from a graph or chart and using these data to solve problems. In the algebra content area, in both 1992 and 2003, students also applied number concepts to explore patterns.

The *Proficient* ALD also states that quantity and spatial relationships should be familiar to students, that they should be able to convey underlying reasoning skills beyond the level of arithmetic, and that they should be able to compare and contrast mathematical ideas and generate examples. There is clear evidence that students performing at the *Proficient* level have a solid command of “quantity” in the sense that they could call upon relevant arithmetic operations and related procedures to solve problems in every content area. In Geometry, students were able to create drawings in 1992 and use spatial skills to solve problems related to folding and reflections in 2003. Also, in both years, students were able to use manipulatives to analyze and compare the areas of shapes. As stated above, students were also able to generate examples or counterexamples in both 1992 and 2003. There was no direct evidence that students could compare and contrast mathematical ideas.

The *Proficient* ALD also states that students should make inferences from data and graphs, apply properties of informal geometry, and accurately use the tools of technology. There was ample evidence in 2003 (but not 1992) that students could make inferences from data and graphs. For example, they could compare and interpret values on a line graph as well as identify and correct an error in statements about a data display. In geometry, students in both years were able to apply properties of informal geometry, such as creating a figure to fit a description, drawing a line of symmetry, or identifying properties of figures. Students at this level could measure with a ruler or protractor and could use a calculator to find the solution to some problems.

The *Proficient* ALD also states that students at this level should understand the process of gathering and organizing data and be able to calculate, evaluate, and communicate results within the domain of statistics and probability. There was no direct evidence that students understand the process of gathering and organizing data. The data had generally already been gathered and organized, but for questions in both 1992 and 2003, students needed to understand the presentation format, whether it was a line graph, a circle graph, a pie chart, or a table. In both years, students were able to solve problems using data read from a graph and to solve probability problems of greater complexity than students at the *Basic* level. In 1992, students were able to apply percentages given in a pie graph, and in 2003, students could recognize bias in a sampling procedure and find the median of a set of values.

***Advanced* Level Analysis**

The introduction to the *Advanced* ALD states that eighth-graders performing at this level should be able to reach beyond the recognition, identification, and application of mathematical rules in order to generalize and synthesize concepts in the five NAEP content areas. In the Number Sense, Properties, and Operations area, only a few items anchored at this level in both 1992 and 2003. Those that did anchor at this level did not yield evidence of the understandings called for by the ALD; none of the items required students to generalize or synthesize concepts. Each of the other four content areas did yield such evidence in both 1992 and 2003. For example, in Measurement, students were able to partition irregular figures to find their area; in Geometry, students could evaluate given information about a figure and draw an additional conclusion about a figure's properties, and in 2003, they could use manipulatives to create, complex shapes with specified properties; in Data Analysis, Statistics, and Probability, students in both years could determine the sample space for a situation involving sampling with replacement, and in 2003, students could solve a multi-step problem that required reading data from one graph and using it in conjunction with data from another graph. In the Algebra and Functions area, in both years they could analyze and explain complex patterns presented in numerical or geometric contexts.

Overall, for the grade 8 assessment, there is excellent alignment between the ALDs and the 1992 and 2003 ADs. In both assessment years, the anchor descriptions of what students knew and could do were very consistent with the policy definitions established for the three achievement levels.

Appendix B

Figure B.1 COMPARISON OF ACHIEVEMENT-LEVEL DESCRIPTIONS AND ANCHOR SUMMARY DESCRIPTIONS FOR GRADE 4

Achievement-Level Description	Anchor Summary Description for 1992	Anchor Summary Description for 2003
There is no description for below the *Basic* level	Below *Basic* Grade 4 (1992) Students performing below the *Basic* level can successfully complete simple routine procedures presented in a familiar way. They can recognize the operations denoted by the symbols +, −, ×, and ÷, and the operation denoted by the phrase 'divided by'. They are able to add and subtract two- and three-digit numbers with one regrouping. These students are also able to recognize common pictorial representations for fractions and can identify measurement instruments for length, temperature, and weight. They can distinguish between appropriate uses of inches and feet. Students performing below the *Basic* level rely on visualization with pictures, not vocabulary, to solve simple geometric questions about shapes and patterns.	Below *Basic* Grade 4 (2003) Students performing below the *Basic* level answer straightforward multiple-choice and simple constructed-response questions requiring well-practiced procedures. These questions typically assess content that was introduced to them in earlier grades. However, there is little evidence that they use mathematics to solve contextual problems. They show evidence of being able to work with small whole numbers and the operations of addition and subtraction, but seem to have little knowledge of multiplication and division. Students at this level can identify a common fraction that represents a shaded region and can recognize common measurement units and instruments for measurement. They can also recognize geometric figures such as circles, triangles, squares, and

Figure B.1 COMPARISON OF ACHIEVEMENT-LEVEL DESCRIPTIONS AND ANCHOR SUMMARY DESCRIPTIONS FOR GRADE 4

Achievement-Level Description	Anchor Summary Description for 1992	Anchor Summary Description for 2003
	Students at this level successfully complete questions with familiar contexts that require beginning routine skills and minimal dependence on mathematical vocabulary. They exhibit limited mathematical decision-making ability and a tenuous grasp of mathematical concepts.	rectangles. In the content area of data and statistics, they can read simple pictographs and bar graphs.
Basic Grade 4 Fourth-grade students performing at the *Basic* level should show some evidence of understanding the mathematical concepts and procedures in the five NAEP content areas. Fourth graders performing at the *Basic* level should be able to estimate and use basic facts to perform simple computations with whole numbers, show some understanding of fractions and decimals, and solve some simple real-world problems in all NAEP content areas. Students at this level should be able to use—	*Basic* Grade 4 (1992) Students performing at the *Basic* level demonstrate a beginning understanding of concepts and emerging skills that are consistent with performance expectations at the fourth-grade level. They are able to identify place value in multi-digit numbers, including tenths, and use understandings of place value in simple problem-solving situations. These students can identify computations needed in the solution of problems and can solve familiar one-step problems presented in a simple	*Basic* Grade 4 (2003) Students performing at the *Basic* level are able to work with larger whole numbers, and show more understanding of place value than students performing below the *Basic* level. They are more adept at computation; addition and subtraction are their strengths, but these students are also beginning to work with multiplication. At this level, students show some ability to utilize comparative reasoning and their problem-solving abilities to solve one-step problems. They possess a beginning conceptual

Figure B.1 COMPARISON OF ACHIEVEMENT-LEVEL DESCRIPTIONS AND ANCHOR SUMMARY DESCRIPTIONS FOR GRADE 4

Achievement-Level Description	Anchor Summary Description for 1992	Anchor Summary Description for 2003
although not always accurately—four-function calculators, rulers, and geometric shapes. Their written responses are often minimal and presented without supporting information.	context. They can add and subtract two- and three-digit numbers with multiple regroupings, and can solve basic multiplication problems. They recognize various situations in which using multiplication is appropriate. Students performing at the *Basic* level display moderate success in answering questions that include either drawings of geometric figures or the manipulation of geometric shapes. For example, students are able to combine triangles and squares into specified figures. They demonstrate an increased growth beyond students below the *Basic* level in working with vocabulary, such as that related to comparative phrases and geometric terms. Students at this level can identify at least one similarity or difference between geometric figures. They exhibit an awareness of appropriate measurement tools and units of measure for length, temperature, and weight.	understanding of length and can use a ruler to measure length to the nearest whole unit. These students use geometric shapes in simple situations, but cannot process multiple conditions within the same problem. However, they are beginning to visualize familiar shapes (in two and three dimensions) and can recognize, extend, and solve simple problems involving patterns in familiar settings. Students at the *Basic* level have greater knowledge of the same content than students performing below the *Basic* level. They are also beginning to demonstrate their ability to use mathematics in questions that do not have as much structure (i.e., in questions that less obviously cue students to use certain algorithms or solution strategies).

Figure B.1 COMPARISON OF ACHIEVEMENT-LEVEL DESCRIPTIONS AND ANCHOR SUMMARY DESCRIPTIONS FOR GRADE 4

Achievement-Level Description	Anchor Summary Description for 1992	Anchor Summary Description for 2003
	They can read common instruments, such as thermometers, in which all scale points are not labeled. These students can read and record data on prepared pictographs and bar graphs. They are also able to identify information presented in tables and graphs and use that information to answer questions involving data, although they cannot interpret data. These students show evidence of a beginning intuitive understanding of probability. Performance at the *Basic* level indicates that students possess a beginning, rather than a firm, knowledge of concepts and acquisition of skills expected of fourth-grade students. They clearly demonstrate growth beyond the below *Basic* performance level in all content areas.	
Proficient Grade 4 Fourth-grade students	*Proficient* Grade 4 (1992) In addition to the	*Proficient* Grade 4 (2003) Students performing at

Figure B.1 COMPARISON OF ACHIEVEMENT-LEVEL DESCRIPTIONS AND ANCHOR SUMMARY DESCRIPTIONS FOR GRADE 4

Achievement-Level Description	Anchor Summary Description for 1992	Anchor Summary Description for 2003
performing at the *Proficient* level should consistently apply integrated procedural knowledge and conceptual understanding to problem solving in the five NAEP content areas. Fourth-graders performing at the *Proficient* level should be able to use whole numbers to estimate, compute, and determine whether results are reasonable. They should have a conceptual understanding of fractions and decimals; be able to solve real-world problems in all NAEP content areas; and use four-function calculators, rulers, and geometric shapes appropriately. Students performing at the *Proficient* level should employ problem-solving strategies such as identifying and using appropriate information. Their written solutions should be organized and presented both with supporting information and with explanations of how they were achieved.	procedural, conceptual, and problem-solving skills exhibited at the *Basic* level, students performing at the *Proficient* level demonstrate greater use and understanding of fourth-grade mathematics, and in more complex situations. For example, they can interpret and solve problems that require two steps or use information from more than one source. Students performing at the *Proficient* level may attain success with a variety of problems presented in word or graphical form that involve operations with whole numbers. They are able to solve problems involving division and interpret remainders in the context of the problem. They can use simple fractions in various ways, including measuring length, interpreting data in pie charts, and identifying simple probabilities. Students at this level show evidence of greater understanding and use of	the *Proficient* level can work with the operation of division in different situations. For example, they can interpret remainders from division in context and use division in a geometry problem to find the length of a side of a geometric figure with equal sides when its perimeter is given. They are developing an understanding of fractions; they can determine the number of unit fractions in a whole, represent simple common fractions and an equivalent fraction for a given fraction using region models, and locate a common fraction on a number line. They are able to solve problems involving elapsed time; for example, they can solve a problem involving half-hour increments in which the change from a.m. to p.m. needs to be factored into the process. These students are expanding their knowledge of mathematical vocabulary and solving problems involving perimeter and area of simple figures.

Figure B.1 COMPARISON OF ACHIEVEMENT-LEVEL DESCRIPTIONS AND ANCHOR SUMMARY DESCRIPTIONS FOR GRADE 4

Achievement-Level Description	Anchor Summary Description for 1992	Anchor Summary Description for 2003
	mathematical concepts and vocabulary, such as "area," "perimeter," and "cube," appearing to be less dependent on being provided with a picture when working with perimeter. They can find the area of simple figures on a grid or by comparison with other figures. They are able to read measurement instruments and axes on graphs that are marked in intervals such as 2s, 5s, or 100s. Students performing at the *Proficient* level show flexibility in their thinking, producing geometric shapes that are described by multiple conditions and using their knowledge of simple figures in less familiar situations. For example, they can find squares of different sizes and orientations in a complex grid. They are able to analyze patterns to solve unfamiliar problems, to extend numerical patterns with constant or decreasing differences, and to see the	They are able to produce partially correct responses to problems that ask them to draw geometric figures that satisfy certain criteria. At this level, students can solve two-step problems and, compared to students at the *Basic* level, are able to visualize geometric figures in more sophisticated settings (for example, in three dimensions). These students are increasing their ability to interpret and create graphs. They demonstrate increased understanding and use of patterns in unfamiliar situations. At the *Proficient* level, students use reasoning in solving problems that require decisions in the solution process.

Figure B.1 COMPARISON OF ACHIEVEMENT-LEVEL DESCRIPTIONS AND ANCHOR SUMMARY DESCRIPTIONS FOR GRADE 4

Achievement-Level Description	Anchor Summary Description for 1992	Anchor Summary Description for 2003
	relationship between two sets of data. They are making progress toward solving more complex, non-routine problems and show some evidence of understanding and partially answering them. Students at this level show they can use a wide spectrum of appropriate fourth-grade mathematical concepts and skills to solve problems in which the tasks are not always inherently obvious.	
Advanced Grade 4 Fourth-grade students performing at the *Advanced* level should apply integrated procedural knowledge and conceptual understanding to complex and nonroutine real-world problem solving in the five NAEP content areas. Fourth graders performing at the *Advanced* level should be able to solve complex nonroutine real-world problems in all NAEP content areas. They should display mastery in	*Advanced* Grade 4 (1992) Students performing at the *Advanced* level can apply their mathematics skills to solve a wide variety of problems and are able to explain their solutions using pictures and words. They can interpret mathematical concepts and language in complex and novel situations, thinking flexibly to solve problems with more than two conditions or steps. At this level, students are able to apply number	*Advanced* Grade 4 (2003) Students performing at the *Advanced* level can solve multi-step problems and can process multiple conditions while solving those problems. They are better able to communicate their processes and actions than students performing at the lower levels are. They can reason more with fractions and show some understanding of decimals to the hundredths place. These students demonstrate the ability to interpret the

Figure B.1 COMPARISON OF ACHIEVEMENT-LEVEL DESCRIPTIONS AND ANCHOR SUMMARY DESCRIPTIONS FOR GRADE 4

Achievement-Level Description	Anchor Summary Description for 1992	Anchor Summary Description for 2003
the use of four-function calculators, rulers, and geometric shapes. The students are expected to draw logical conclusions and justify answers and solution processes by explaining why, as well as how, they were achieved. They should go beyond the obvious in their interpretations and be able to communicate their thoughts clearly and concisely.	skills and concepts to solve problems in a variety of contexts. They have a more developed understanding of fractions and can use fraction concepts and skills in measurement and probability situations. These students can use their understanding of area and perimeter to solve problems. They are able to use rulers accurately, can apply strong visualization skills to solve problems involving more complex geometric figures, and can recognize and apply geometric terms and relationships in problem situations. For example, they can construct a geometric figure that meets multiple criteria or visualize the characteristics of a cube. At the *Advanced* level, students can interpret probability situations and can determine the number of possible outcomes for a simple event. They show evidence of using more sophisticated mathematics such as that	mathematics in more novel situations and show growth in their mathematical language. They demonstrate the ability to use proportional reasoning to solve simple problems and show an initial understanding of functions. Across all of the content areas, students at the *Advanced* level are able to extend the use of mathematics to less familiar and more complex situations.

Figure B.1 COMPARISON OF ACHIEVEMENT-LEVEL DESCRIPTIONS AND ANCHOR SUMMARY DESCRIPTIONS FOR GRADE 4

Achievement-Level Description	Anchor Summary Description for 1992	Anchor Summary Description for 2003
	in problems involving rates. Performance at the *Advanced* level indicates that the students possess a deeper understanding of mathematical content and a greater ability to apply and communicate that understanding in a variety of situations.	

Figure B.2 COMPARISON OF ACHIEVEMENT-LEVEL DESCRIPTIONS AND ANCHOR SUMMARY DESCRIPTIONS FOR GRADE 8

Achievement-Level Description	Anchor Summary Description for 1992	Anchor Summary Description for 2003
There is no description for below the *Basic* level	Below *Basic* Grade 8 (1992) Performance on items that anchor below the *Basic* level shows that students performing below *Basic* can do simple computations, carry out straightforward measurement tasks, and show evidence of a beginning understanding of selected topics in geometry, data, and pre-algebra. In the area of Numbers and Operations, students are able to perform arithmetic operations without the use of a calculator with up to three-digit numbers with and without regrouping. They can recognize and determine decimal values to the tenths place when represented in different formats such as a number line. They are able to solve one-step and simple two-step word problems involving operations with money and in other straightforward contexts. They are able to identify a shaded rectangular region that represents a	Below *Basic* Grade 8 (2003) Performance on items that anchor below the *Basic* level reveals that students can solve simple word problems; can compute with whole numbers; can solve simple measurement and geometry problems involving weight, estimation, and shapes; can understand and solve problems involving simple graphs; and can perform simple algebraic operations. All unit-related measurement questions that anchored at this level involved conventional units of measure as opposed to metric units. Students at this level had some success on short answer constructed-response questions, but virtually no success on extended constructed-response questions. In the area of Number Sense, Properties, and Operations, students are able to compute with whole numbers without remainders, in both multiple choice and free

Figure B.2 COMPARISON OF ACHIEVEMENT-LEVEL DESCRIPTIONS AND ANCHOR SUMMARY DESCRIPTIONS FOR GRADE 8

Achievement-Level Description	Anchor Summary Description for 1992	Anchor Summary Description for 2003
	given common fraction, use estimation appropriately in simple problems, and display some understanding of multiples of one-digit whole numbers. They can recognize a six-digit numeral, given its verbal form in a context. In the area of Measurement, students demonstrate some understanding of length in metric and customary units by measuring a distance, using a standard conversion relationship, and determining the reasonableness of units. They demonstrate some understanding of weight in customary units by reading analog scales and comparing weights of common objects. They can find or compare the areas of simple irregular figures using unit squares and determine an appropriate instrument for measuring a given attribute. In the area of Geometry, students are able to use given cardboard geometric shapes to form simple composite figures	response formats and can use concepts of place value with whole numbers, including rounding and estimating. They can solve simple word problems involving whole number operations and money and are able to recognize and identify area representations for common and equivalent fractions. In the area of Measurement, students can read a scale, identify the coordinate of a point on a number line, compare weights, and identify appropriate units for area. They can find areas using estimation or a background grid and can visually compare the size of angles. They can convert from one conventional unit to another when the conversion factor is given. If given the perimeter of a square, they can find the length of a side. They can compare the capacity of rectangular solids, given the three dimensions. In the area of Geometry and Spatial Sense, students are able to

Figure B.2 COMPARISON OF ACHIEVEMENT-LEVEL DESCRIPTIONS AND ANCHOR SUMMARY DESCRIPTIONS FOR GRADE 8

Achievement-Level Description	Anchor Summary Description for 1992	Anchor Summary Description for 2003
	and use given diagrams and figures to solve spatial visualization problems involving simple transformations and three-dimensional shapes. They can identify and distinguish between both simple two-dimensional figures and common three-dimensional objects. In the area of Data, Statistics, and Probability, students can use given information to create simple data displays such as pictographs and bar graphs. They can draw a simple conclusion from information presented in a pie chart. Given a simple probability situation, students can determine if an event is possible. In the area of Algebra, students can choose the next term in a given visual pattern, evaluate a simple numerical expression using the order of operations, and solve for a variable represented by a box in a simple number sentence. They can select an expression using a variable represented by a	identify or draw the reflection of a simple shape through a horizontal or vertical line. They can recognize cylinders in various orientations and can identify two-dimensional figures that can be folded into simple three-dimensional shapes. They can choose a counter-example to a given statement about quadrilaterals. In the area of Data Analysis, Statistics, and Probability, students can read simple bar graphs, pie charts, and pictographs. They can solve simple one-step word problems involving data displays and can use proportional reasoning to solve simple problems involving data. In the area of Algebra and Functions, students can solve simple one- and two-step equations involving small whole numbers as well as word problems involving two variables with small whole numbers. They can read points on a grid, including a coordinate plane and map. They can

Figure B.2 COMPARISON OF ACHIEVEMENT-LEVEL DESCRIPTIONS AND ANCHOR SUMMARY DESCRIPTIONS FOR GRADE 8

Achievement-Level Description	Anchor Summary Description for 1992	Anchor Summary Description for 2003
	box to describe the information given in a one-step word problem.	compute using parentheses to indicate order of operations. They can extend or find terms in a visual pattern. They can recognize a solution to a one-step inequality in one variable or identify an expression that represents a given situation.
Basic Grade 8 Eighth-grade students performing at the *Basic* level should exhibit evidence of conceptual and procedural understanding in the five NAEP content areas. This level of performance signifies an understanding of arithmetic operations—including estimation—on whole numbers, decimals, fractions, and percents. Eighth-graders performing at the *Basic* level should complete problems correctly with the help of structural prompts such as diagrams, charts, and graphs. They should be able to solve problems in all NAEP content areas through the appropriate	*Basic* Grade 8 (1992) Students at the *Basic* level can work with common fractions, find the area and perimeter of a rectangle, and draw simple geometric figures. They can solve simple probability problems and recognize the solution to a linear equation. They display greater problem-solving ability than do students below the *Basic* level. Students could use more complex diagrams and figures at this level than students below the *Basic* level. Also, they could successfully respond to items with less elementary language. As was the case for students performing below the *Basic* level, students at the *Basic* level had very	*Basic* Grade 8 (2003) Students at the *Basic* level can solve a variety of one-step word problems, including those that involve simple percent. They can use rulers and manipulatives to demonstrate understanding of measurement and geometry concepts. They demonstrate a beginning understanding of a variety of statistical concepts related to the use of data displays and sampling. In the area of Algebra, students performing at the *Basic* level are able to do considerably more than students performing below the *Basic* level. The diagrams and terminology used in the presentation of problems

Figure B.2 COMPARISON OF ACHIEVEMENT-LEVEL DESCRIPTIONS AND ANCHOR SUMMARY DESCRIPTIONS FOR GRADE 8

Achievement-Level Description	Anchor Summary Description for 1992	Anchor Summary Description for 2003
selection and use of strategies and technological tools—including calculators, computers, and geometric shapes. Students at this level also should be able to use fundamental algebraic and informal geometric concepts in problem solving. As they approach the *Proficient* level, students at the *Basic* level should be able to determine which of the available data are necessary and sufficient for correct solutions and use them in problem solving. However, these eighth-graders show limited skill in communicating mathematically.	limited success on extended constructed-response questions. In the area of Numbers and Operations, students are able to recognize equivalent fractions in a variety of representations, such as shaded rectangular regions and number lines. They can solve one-step word problems involving addition, multiplication, and division with simple mixed numbers and common fractions. They are able to solve one-step and simple two-step word problems involving division of two- and three-digit whole numbers, and interpret the remainder. Students can solve multi-step word problems involving operations with money. They can use place value concepts in problem-solving contexts. For example, students can compare possible values for a set of digits and identify a number that rounds to a given number. Students can explain or provide an example to show that multiplying a positive one-digit number by	at this level are somewhat more advanced than for items that anchored below this level. As was the case for students below the *Basic* level, students at this level had very limited success in answering extended constructed-response questions. In the area of Number Sense, Properties, and Operations, students can solve one-step word problems involving simple percent, interpretation of remainders, and division of fractions with a whole-number quotient, both with and without the use of a calculator. They have limited success in solving multi-step problems, but can identify extraneous information in a word problem. They can represent fractions using area and number-line models and can identify a representation of equivalent fractions. They show a beginning understanding of proportional reasoning by using percents, comparing rates in context, and identifying equivalent ratios. They can use place value

Figure B.2 COMPARISON OF ACHIEVEMENT-LEVEL DESCRIPTIONS AND ANCHOR SUMMARY DESCRIPTIONS FOR GRADE 8

Achievement-Level Description	Anchor Summary Description for 1992	Anchor Summary Description for 2003
	another number can result in a number less than the original number. In the area of Measurement, students can determine length in metric and customary units, including using a ruler in a nonstandard position. They can solve problems involving the perimeter and area of rectangles, including finding missing dimensions. Students can use diagrams and related information to solve problems involving dials and balance scales. In the area of Geometry, students can draw simple geometric figures given one or more conditions. They are able to solve problems involving two-dimensional figures that can be folded to form a cube. They can compare the measure of an angle to a right angle and identify the reflection of a point located on a geometric figure. In the area of Data, Statistics, and Probability, students can solve simple probability problems, including listing a sample	concepts with decimals to the hundredths. In the area of Measurement, students can use aspects of proportional reasoning to solve one-step word problems involving weight or time. They can interpret scales and readings on measurement devices and can identify coordinates of mixed numbers and common fractions on a number line. They can use a ruler, or a picture of a ruler, to measure or to draw a geometric figure with dimensions given in conventional units. They can both identify and draw angles that are greater than or smaller than a right angle. Given the measure of two angles of a triangle they can find the measure of the third angle. In the area of Geometry and Spatial Sense, students are able to partition a simple or complex shape with or without manipulatives. They demonstrate an understanding of reflections, rotations, and

Figure B.2 COMPARISON OF ACHIEVEMENT-LEVEL DESCRIPTIONS AND ANCHOR SUMMARY DESCRIPTIONS FOR GRADE 8

Achievement-Level Description	Anchor Summary Description for 1992	Anchor Summary Description for 2003
	space. Given a description of a survey, students can explain whether a particular sampling method is appropriate. Students can draw conclusions about data displayed in tables, line graphs, and bar graphs. In the area of Algebra, students can use a given list of values, a picture, or a verbal description of a simple pattern to find missing terms or extend the pattern to the next term. Students can recognize the solution to a variety of equations, such as a simple linear equation with two variables, an equation involving a square root, and an equation represented pictorially. They can find possible solutions to simple inequalities with a variable represented by a box and understand that a variable can take on a number of values in an expression. They can identify an algebraic expression that represents the verbal description of a problem.	symmetry with or without the use of manipulatives. They can use spatial visualization with manipulatives to identify the possible shape when two common figures overlap. They can solve a standard problem involving side lengths of similar triangles. In the area of Data Analysis, Statistics, and Probability, students can read data from a stem-and-leaf plot, solve simple word problems involving probability, identify the median of a set of values that is odd in number, and determine the truth of statements about given data displays. They understand the effect of sample size in designing a survey. In the area of Algebra and Functions, students can complete a number pattern and write the rule. They can also complete a nonnumeric pattern, given selected terms and conditions. Students can identify the coordinates of a vertex of common geometric figures not in the first quadrant, given

Figure B.2 COMPARISON OF ACHIEVEMENT-LEVEL DESCRIPTIONS AND ANCHOR SUMMARY DESCRIPTIONS FOR GRADE 8

Achievement-Level Description	Anchor Summary Description for 1992	Anchor Summary Description for 2003
		the coordinates of the other vertices. Students understand the meaning of the square root symbol, can identify the solution to a pictorial equation, and can compute a result of a numerical expression without parentheses, using order of operations. Finally, they can identify the expression for a relationship among three variables in context.
Proficient Grade 8 Eighth-grade students performing at the *Proficient* level should apply mathematical concepts and procedures consistently to complex problems in the five NAEP content areas. Eighth-graders performing at the *Proficient* level should be able to conjecture, defend their ideas, and give supporting examples. They should understand the connections among fractions, percents, decimals, and other mathematical topics such as algebra and functions. Students at this level are expected to have a	*Proficient* Grade 8 (1992) Items that anchored at this level showed a jump in the demand level of reasoning and problem-solving required. Students had greater success than those performing below *Proficient* in answering extended constructed-response questions. They can solve problems involving percentages, apply proportional reasoning concepts, identify properties associated with geometric figures, and solve nonroutine data-related problems. In the area of Numbers and Operations, students	*Proficient* Grade 8 (2003) Students at the *Proficient* level can solve somewhat more complex problems in all five content areas than students at the *Basic* level and below. They can analyze and explain problem situations, can deal with increasingly complex language and figures, can extend patterns, and can apply spatial visualization skills. While measurement items involving units that anchored below the *Proficient* level generally used conventional units of measure, students at the *Proficient* level can also solve measurement and geometry problems

Figure B.2 COMPARISON OF ACHIEVEMENT-LEVEL DESCRIPTIONS AND ANCHOR SUMMARY DESCRIPTIONS FOR GRADE 8

Achievement-Level Description	Anchor Summary Description for 1992	Anchor Summary Description for 2003
thorough understanding of basic-level arithmetic operations—an understanding sufficient for problem solving in practical situations. Quantity and spatial relationships in problem solving and reasoning should be familiar to them, and they should be able to convey underlying reasoning skills beyond the level of arithmetic. They should be able to compare and contrast mathematical ideas and generate their own examples. These students should make inferences from data and graphs, apply properties of informal geometry, and accurately use the tools of technology. Students at this level should understand the process of gathering and organizing data and be able to calculate, evaluate, and communicate results within the domain of statistics and probability.	can solve one- and two-step word problems involving finding the percent of a number. They can also solve one- and two-step word problems involving proportional reasoning in a variety of mathematical contexts. They are able to use reasoning and problem solving to generate examples and counter examples involving number relationships, for example, identifying a counterexample in a problem involving even and odd numbers or writing a word problem to fit a given context. They can order fractions with unlike one-digit denominators and follow a series of written directions involving arithmetic operations on whole numbers and fractions. They can also work with negative numbers. In the area of Measurement, students can use a protractor to find the degree measure of a given angle. They can use cardboard shapes to compare the areas of common	that use metric measures. Students at this level are partially successful in solving and explaining questions that require an extended response. In the area of Number Sense, Properties, and Operations, students are able to work with large and small numbers by identifying the expanded form of a number written in scientific notation, by solving a one-step word problem involving a number of millions expressed in decimal form, and by identifying the number that corresponds to a point on a number line between two given points to the thousandths place. They can identify fractions that are correctly ordered relative to a benchmark fraction. At this level, they can solve two-step word problems involving simple percent, decimals to tenths, whole numbers, and proportional reasoning. They can explain the reasoning used to solve some word problems and can choose a number that makes a statement about even

Figure B.2 COMPARISON OF ACHIEVEMENT-LEVEL DESCRIPTIONS AND ANCHOR SUMMARY DESCRIPTIONS FOR GRADE 8

Achievement-Level Description	Anchor Summary Description for 1992	Anchor Summary Description for 2003
	geometric figures and explain the results. They can analyze irregular polygons to determine possible perimeters and can identify a numerical expression for the area of a rectangle with given dimensions. They are able to solve a multi-step word problem involving rounding and a given conversion to determine the area of a rectangle in customary units. In the area of Geometry, students can solve problems involving interior and exterior angle measures in triangles. Given pictures and/or information about common geometric figures such as circles, parallelograms, or triangles, students are able to identify correct and incorrect statements about their properties. They can create a drawing to fit a verbal description of points on a line and also draw a line of symmetry for geometric figures. In the area of Data, Statistics, and Probability, students can compare the amount of change	and odd numbers false. In the area of Measurement, students can use a ruler to draw nested rectangles with given conditions, use manipulatives to analyze and compare areas, and find the measure of a missing acute angle in a right triangle. They can also answer questions that demonstrate an understanding of area and perimeter relative to a rectangle. In the area of Geometry and Spatial Sense, students are able to use spatial visualization to identify the result of folding paper, identify the reflection of a point in a nonstandard context, recognize properties of a given quadrilateral, and identify the proper name for a triangle, given a figure and the lengths of its sides. They can also determine the length of the hypotenuse of a right triangle, given the lengths of its legs. In the area of Data Analysis, Statistics, and Probability, students can recognize bias in a

Figure B.2 COMPARISON OF ACHIEVEMENT-LEVEL DESCRIPTIONS AND ANCHOR SUMMARY DESCRIPTIONS FOR GRADE 8

Achievement-Level Description	Anchor Summary Description for 1992	Anchor Summary Description for 2003
	between various intervals on a graph. They can identify the solution to a two-step probability problem. They can also identify sets of data that have a given average. They are able to use percents given in a pie chart to find the amount a specific section represents. In the area of Algebra, students are able to identify possible ordered pair solutions to linear equations in two variables. Given a table showing a pattern, students can analyze the pattern to determine a term later in the pattern. Students can use negative numbers in a variety of contexts, including the graphing of inequalities on a number line. Students are able to locate a point in a coordinate plane, with or without grids marked, given directional orientation or geometric relationships.	sample, identify the median of a set of values that is even in number, and compare and interpret values on a line graph. They can identify and correct an error in statements about a data display and can solve word problems involving data displays, including pie graphs with percents. Finally, they can identify the number of events that will produce a given probability. In the area of Algebra and Functions, students can identify a solution to a word problem involving two variables and recognize the possible solutions to a two-step inequality in one variable and the solution to a one-step equation involving two variables. They can graph a compound inequality on a number line and also analyze and extend nonroutine numeric patterns. They can identify the equations for patterns given in table form. Given two coordinate points not shown, they can identify a true statement about the line these points determine. They can

Figure B.2 COMPARISON OF ACHIEVEMENT-LEVEL DESCRIPTIONS AND ANCHOR SUMMARY DESCRIPTIONS FOR GRADE 8

Achievement-Level Description	Anchor Summary Description for 1992	Anchor Summary Description for 2003
		identify possible solutions to a multi-step logic problem and estimate a point of intersection in a coordinate graph. They can apply formulas given in equation form or in words. They can also apply the distributive property to a variable expression.
Advanced Grade 8 Eighth-grade students performing at the *Advanced* level should be able to reach beyond the recognition, identification, and application of mathematical rules to generalize and synthesize concepts and principles in the five NAEP content areas. Eighth-graders performing at the *Advanced* level should be able to probe examples and counterexamples to shape generalizations from which they can develop models. Eighth-graders performing at the *Advanced* level should use number sense and geometric awareness to consider the reasonableness of an	*Advanced* Grade 8 (1992) Students performing at the *Advanced* level can integrate and apply their mathematics skills to solve a wide range of problems in the various content areas. They attain a higher level of performance on extended constructed-response questions than do students at the proficient level. In the area of Numbers and Operations, students are able to analyze problems containing multiple conditions involving money, time, and operations on even, odd, or consecutive integers where multiple solutions are required. Given a number in scientific notation	*Advanced* Grade 8 (2003) Students at the *Advanced* level can solve a greater variety of nonroutine, multi-step problems in various content areas than can students at the *Proficient* level and below. They also exhibit a greater command of the language of mathematics and can successfully deal with increasingly complex figures and problem situations. They can use spatial visualization with manipulatives to create complex shapes and identify properties of figures and classes of figures. Students at this level can successfully solve a variety of extended constructed-response problems, which was not the case for students performing at

Figure B.2 COMPARISON OF ACHIEVEMENT-LEVEL DESCRIPTIONS AND ANCHOR SUMMARY DESCRIPTIONS FOR GRADE 8

Achievement-Level Description	Anchor Summary Description for 1992	Anchor Summary Description for 2003
answer. They are expected to use abstract thinking to create unique problem-solving techniques and explain the reasoning processes underlying their conclusions.	displayed on a calculator, students can identify the equivalent decimal representation. In the area of Measurement, students demonstrate an understanding of the concept of accuracy of measurement in the context of length. Students can measure the perimeter of a geometric figure using nonstandard units. They can determine the surface area of a rectangular solid composed of cubes shown in a perspective drawing. Students can use the concept of partitioning to solve multi-step word problems involving areas of rectangles, circles, and irregular rectilinear regions. In the area of Geometry, students can use the Pythagorean theorem. They can use given information about an unspecified quadrilateral to draw an additional conclusion about the figure's properties. They can solve problems relating central angles of	the *Proficient* level and below. In the area of Number Sense, Properties, and Operations, students demonstrate understanding of properties of integers, can interpret the conditions of a multi-step word problem and provide the relevant computations, and can compare and explain a multi-step word problem involving additive and successive percent discounting. In the area of Measurement, students can determine area in a variety of contexts, such as the surface area of a solid, tiling, and partitioning. They can use a protractor to draw a directional angle and solve a nonroutine problem involving arc length and estimation in context. Finally, given a measurement and a level of accuracy, students can find a possible length. In the area of Geometry and Spatial Sense, students are able to use spatial visualization with

Figure B.2 COMPARISON OF ACHIEVEMENT-LEVEL DESCRIPTIONS AND ANCHOR SUMMARY DESCRIPTIONS FOR GRADE 8

Achievement-Level Description	Anchor Summary Description for 1992	Anchor Summary Description for 2003
	circles to sectors and arc lengths. Given a line and three points on the line, students can identify the description of a construction that would yield a familiar angle. In the area of Data, Statistics, and Probability, students can determine the median in different situations, such as given information in a list or scatter plot. Given a frequency distribution table, students are able to determine the average. Students can analyze and solve multi-step probability problems. They demonstrate understanding of the relationship between the sector of a circle and its degree measure. In the area of Algebra, students can analyze and explain complex patterns presented in numerical or visual contexts. Students can evaluate a numerical expression involving a variety of operations, including exponents, using the conventional order of operations.	manipulatives to create complex shapes with specified properties. They can relate central angles to arc lengths as well as relate the dimensions of two common shapes, a sphere and a cylinder, in a real-world context. They can identify properties of quadrilaterals and understand the relationship among classes of quadrilaterals. Given an interior and an exterior angle of a triangle, they can find other angles. In the area of Data Analysis, Statistics, and Probability, students can evaluate statements about data presented in a pie graph, determine the favorable outcomes and the sample space for probability situations including sampling without replacement, and find a weighted average. They can solve multi-step word problems such as reading data from one bar graph and using it in conjunction with data from another bar graph. In the area of Algebra and Functions, students

Figure B.2 COMPARISON OF ACHIEVEMENT-LEVEL DESCRIPTIONS AND ANCHOR SUMMARY DESCRIPTIONS FOR GRADE 8

Achievement-Level Description	Anchor Summary Description for 1992	Anchor Summary Description for 2003
		can solve word problems involving two variables, verify the extension of patterns in both numerical and geometric contexts, and find an approximate solution to a word problem by extending two lines to their point of intersection. Given a linear equation and a change in one of the variables, students can identify the effect on the other variable. They can also identify the graph of a given linear equation. They can identify equivalent expressions given in verbal form and also identify the solution to a nonroutine multi-step word problem.