



**Technical Report on the SAT/NAEP Grade 12 Preliminary Comparability Study—
Mathematics¹**

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Background

Purpose

The purpose of this study was to examine the overall comparability of the SAT mathematics section and the NAEP grade 12 mathematics assessment. The scope of this preliminary examination of comparability was to determine feasibility prior to funding a more extensive and rigorous alignment study. If NAEP assessments and the SAT sections are judged to be comparable at a content specifications level, it may be reasonable to move forward with an extensive study that examines alignment at the item and scale levels.

Investigation of the alignment of NAEP and the SAT is part of an effort to determine ways to evaluate how grade 12 NAEP can be used to report students' preparedness for post-secondary activities. Content alignment studies are recommended as a first step to other studies that relate performance on another assessment to NAEP.

Comparability

A key question, however, is what constitutes comparability—how comparability is to be defined, conceptually and operationally. Issues of comparability, to date, primarily have focused on gathering judgment-based evidence regarding how well tests align with content standards (e.g., Bhola, Impara, & Buckendahl, 2003; Webb, 1997; Webb, Herman, & Webb, 2007). Similar methods and criteria for alignment, however, may be applied to a test-to-test alignment investigation. In this context, two different levels of comparability may be considered.

- The first, at the test content specification level, addresses the basic question: “Do the two tests measure the same content areas within the given subject?”
- The second, at the level of the individual items, focuses on the cognitive demands of the items: “Do the items on the two tests require the same depth of content understanding?”

For this preliminary study, the first question is examined—the focus is on NAEP and SAT frameworks or specifications. On the basis of the results, the College Board and the National Assessment Governing Board may proceed to a more extensive study to address the second question.

Design

Panelists

Evaluation of comparability was based on expert judgment. A panel of mathematics experts drawn from the past or current SAT and NAEP standing content committees was assembled. Shown in table 1 are the characteristics of the panel members. Table 2 compares the characteristics of the panel members with those of the NAEP and SAT standing committees. The proportion of panel members in each occupational setting (K-12 vs. post-secondary) is weighted toward the K-12 side, reflecting in part the fact that one or more committee members who were invited—who would have additionally

represented post-secondary—could not attend. It should also be acknowledged that Panelist 5 has had experience in the post-secondary setting. In terms of region, the only region not represented on the panel was the northeast. It should be noted, however, that one of the panelists was from Maryland but was coded as being from the south as per Census region guidelines. Therefore, the panel is somewhat more diverse geographically than the region data suggest.

Descriptions of Content Areas

SAT Mathematics

For the SAT Mathematics section, performance characteristic descriptors were used as an outline for the content in the section. As shown in table 3, there are nine content areas in the assessment: (A) Numbers and Operations, (B) Algebra and Functions, (C) Geometry and Measurement, (D) Data, Statistics, and Probability, (E) Problem Solving, (F) Representation, (G) Reasoning, (H) Connections, and (I) Communication. There are in turn 125 descriptors spread across these content areas.

These descriptors are “detailed explanations of what students within specific SAT score intervals know and can do as indicated by their performance on the SAT” (College Board 2007). They were derived by looking at the types of items that students within given intervals (such as 200 to 290, 300 to 390, and so on) answered correctly, and then developing descriptions of the knowledge skills needed to answer them. For the purpose of this study, the descriptors were grouped by content area, with descriptors for lower score levels preceding those from higher ones (numeric score bands were not identified). The SAT Mathematics performance category descriptors can be found in Appendix A.

NAEP Grade 12 Mathematics

For the NAEP Grade 12 Mathematics Assessment, the objectives from the framework served as the list of content covered by the test. As shown in table 4, the 130 objectives are grouped into five content areas: (A) Number Properties and Operations, (B) Measurement, (C) Geometry, (D) Data Analysis, Statistics, and Probability, and (E) Algebra. The NAEP objectives are included in Appendix B.

Ratings

Test-level comparability addresses the extent to which the content areas covered by the SAT mathematics test are addressed by the content areas in the NAEP grade 12 mathematics assessment. Panelists were asked to make two basic kinds of ratings: (1) content-area ratings and (2) overall ratings.

Content-Area Ratings

After reviewing the SAT and NAEP descriptors², the panelists were asked to do the following:

- Refer to the NAEP descriptors to see whether the SAT descriptor is covered by NAEP. For example, if an SAT mathematics performance characteristic descriptor states that students should be able to “identify factors of whole numbers,” is there a NAEP grade 12 mathematics objective that covers the same content? This judgment was made using a “Yes” or “No” rating.
- For those “matched” content categories for which the panelist responded “yes,” the panelist was asked to do the following:
 - Indicate the strength of the NAEP-SAT comparability for that SAT descriptor. This judgment was made using a 3-point rating scale, ranging from (1) weak to (2) moderate to (3) strong.
 - Identify the specific NAEP descriptor(s) that cover the SAT descriptor.

Overall Ratings

The panelists were also asked to make three overall ratings.

- 1) The first was a judgment of whether the content of the two tests is comparable, using a 4-point scale ranging from Strongly Agree to Strongly Disagree.
- 2) The second was a judgment of whether the overall breadth of mathematics skills on the two tests is comparable, using a 4-point scale ranging from Strongly Agree to Strongly Disagree.
- 3) The final was a Yes or No judgment of whether, based on the panelist’s evaluation of comparability, he or she thinks that a follow-up study comparing items on each test is justified.

Meeting Procedures

The meeting was conducted on Monday, February 4, 2008, with five panelists, six staff from ETS (facilitators and test development staff), and one representative from the National Assessment Governing Board. The agenda for the meeting appears in table 5.

Welcome, Introductions, and Background Overview

Following a general introduction and welcome, all participants were asked to introduce themselves, and the agenda for the day was reviewed. A representative for the National Assessment Governing Board then provided background for the study; the Governing Board presentation is included in Appendix C.

² For the purpose of this report, the term *descriptor* will be used as shorthand for both the SAT performance characteristic descriptors and NAEP objectives.

Content Overviews

Next, one ETS test development staff member provided an overview of the SAT assessment, and another provided an overview of the NAEP assessment. For the SAT, the purpose of the mathematics section was described, followed by a review of its features. The structure of the mathematics section was then outlined, including the timing of the test and the number of items. The calculator policy was also reviewed. The difference between a reasoning test such as the SAT and an achievement test was also described; it was noted that questions are difficult not because they assess more advanced mathematical content (e.g., precalculus and beyond), but because they require synthesis and application of more elementary concepts. Finally, each panelist was given a copy of a sample SAT mathematics section from the 2007–2008 SAT Preparation Booklet. (This booklet was also sent to panelists in advance of the meeting.) A copy of the SAT presentation is included in Appendix D.

For the NAEP 12th-grade mathematics assessment, a description of the purpose of the test was given, along with a review of its features. The structure of the NAEP mathematics test, a survey assessment, was then described. A description was given of ways in which performance on the assessment was reported, both through average scores and through the achievement levels that are used to categorize groups of students. Information on the number of items and the time given to each student was provided. Both the content areas (described above) and levels of complexity (low, moderate, or high), which are a measure of the cognitive demands an item makes on the student, were reviewed. Lastly, each panelist was given a copy of the pre-publication edition of the Framework for the 2009 NAEP Mathematics Assessment (2007), as well as sample blocks from the 2008 NAEP 12th-grade mathematics pilot assessment (one calculator block and one non-calculator block were provided). (The pre-publication framework was also sent to panelists in advance of the meeting.) A copy of the NAEP presentation is included in Appendix E.

Training in Content-Area Ratings

The panelists were then trained to make content-area comparability judgments. As described above, the first judgment was whether the SAT descriptor is covered by one or more NAEP descriptors, using a “Yes” or “No” rating. For those SAT descriptors for which the panelist responded “Yes,” he or she was asked to (a) indicate the strength of the NAEP-SAT comparability for that SAT descriptor, using a 3-point rating scale, ranging from (1) weak to (2) moderate to (3) strong, and to (b) identify the specific NAEP descriptor(s) that cover the SAT descriptor. A copy of the process training slides is contained in Appendix F.

Once the explanation of the process of making the content-area judgments was completed, the panelists were asked to read through both the SAT and NAEP descriptors. The panelists were then asked to provide ratings for the first three SAT mathematics descriptors. Panelists had the opportunity to discuss these initial judgments, helping them to come to a shared understanding of the judgment process. A copy of the content-area rating form is contained in Appendix G.

First Evaluation Form

Panelists were next given the Initial Evaluation/Ready-to-Proceed form. The panelists were asked to provide feedback on the extent to which they understood the purpose of the study, the degree to which the overview of the assessment was presented clearly, how clearly the steps to be followed in making ratings were presented, and the degree to which they understood what was expected of them in completing content-area ratings. They were also asked to sign off on whether they were ready to proceed to make the first set of content-area comparability judgments. A copy of the initial evaluation form is included in Appendix H.

Round 1 Ratings

After it was confirmed that all panelists felt ready to proceed, they were asked to complete the remaining round 1 ratings. It was anticipated that the ratings would take approximately two hours to complete. However, at noon, when it was anticipated that ratings would be ready to be entered into the spreadsheet, panelists were still working, and none had completed ratings for half of the 125 descriptors.³ Therefore, after breaking for lunch, panelists were asked to return to their ratings. Two panelists—Panelists 2 (SAT committee) and 3 (NAEP committee)—were asked to stop where they were and then to start again at the end of the descriptors, working backwards. This decision was made in consultation with the Governing Board staff member so that if time did not allow for all round 1 ratings to be completed, every descriptor would have at least two ratings completed for it.

Feedback and Discussion

The entry of round 1 ratings could not be completed in time for ratings for specific descriptors to be flagged and discussed. Instead, panelists were asked to discuss the content areas that they found most difficult, and a few items within those content areas.

Preparation for Round 2 Ratings

Round 1 ratings were entered as the discussion took place. The rating forms were then returned to the panelists so that they could take them home to complete round 2 ratings.

Post-Meeting Procedures

Summary of Round 1 Ratings

Immediately after the meeting ended, additional information was added to the spreadsheet containing round 1 ratings. For every objective for which someone had entered a “No” rating, the objectives noted by others who gave the item a “Yes” rating were entered into a separate column. This was done to enable panelists who did not think the content of a given SAT descriptor was covered by a NAEP objective the

³ The pace of the ratings was affected by several factors. First, the presentation of information about the assessments took longer than planned, which caused the start of round 1 ratings to be delayed. Second, the number of SAT descriptors for mathematics was twice as large as that for critical reading; therefore, the round 1 ratings could not be completed in the time allotted in the agenda, which had been the same for both reading and mathematics.

chance to look at the objectives other panelists thought were relevant. (Each “No” rating provided by every panelist was also highlighted.)

A separate Excel file was then created for each panelist so that he or she would see the other panelists’ ratings for round 1, but would have clearly marked columns in which to enter the round 2 ratings. A worksheet was also included in which the panelists could provide overall ratings. These were e-mailed to the panelists the night of the meeting.

Round 2 Ratings

Panelists were asked to record revised ratings in the Excel file based on review of their ratings as well as those of other panelists. They were told that they were not required to revise any ratings if they did not wish to do so. Panelists returned those ratings by e-mail over the next several days.

Overall Ratings

Panelists were asked to complete their overall ratings after completing their round 2 ratings. As described above, the ratings focused on the degree to which panelists believed that the tests covered the same type and the same range/breadth of mathematics skills, and whether a follow-up study comparing items on each test is justified. A copy of the overall rating form, which was provided to panelists in an Excel file following the meeting, is contained in Appendix I.

Final Evaluation

Panelists were asked to complete the final evaluation form electronically. First, the panelists were asked whether the SAT and NAEP descriptors were sufficiently detailed to judge comparability. Next, they were asked several questions about the content-area comparability ratings—whether the rating form was easy to complete, whether the summary of ratings was presented clearly, whether the discussion of the summary of the ratings was informative, and whether the process of completing the ratings was easy to follow. Finally, the panelists were asked a parallel set of questions about the overall ratings. A copy of the final evaluation form can be found in Appendix J.

Results

Content-Level Ratings

Summary-Level Results

Tables 6 and 7 present summaries of the ratings provided for each round for each content area. The total number and percentage of No, Yes (1), Yes (2), Yes (3), and blank ratings are given. For the Yes ratings, Yes (1) indicates weak comparability; Yes (2) indicates moderate comparability; and Yes (3) indicates strong comparability.

Figure 1 graphically presents the information for round 1. Content area H (Connections) had the highest percentage of No ratings at 37 percent, and content areas A (Number and Operations) and D (Data, Statistics, and Probability) had the lowest at 9 percent. For

Yes (3) ratings, content area F (Representation) had the highest percentage at 43 percent, and content area E (Problem Solving) the lowest at 13 percent. For blank ratings, content area D (Data, Statistics, and Probability) had the highest percentage at 24 percent, and content area F (Representation) had the lowest at 1 percent.

Data for round 2 ratings are graphed in figure 2. The highest percentage of No ratings is now found in content area G (Reasoning) at 28 percent. The lowest percentage of No ratings is now found for F at 3 percent. For Yes (3) ratings, the highest percentage is now found for content area D at 51 percent, while the lowest percentage is again found for content area E (staying at 13 percent). For blank ratings, content area I (Inferencing) has the highest percentage at 7 percent, and content areas G and H (Connections) have the lowest at 0 percent.

The differences across rounds can be more easily seen in figure 3. The percentage of No ratings increased in every content area, ranging from a 2 percent increase for content area D to a 13 percent increase for content area H. The largest increases for Yes (1) ratings were found in content area G, with a 14 percent increase. Increases were found in the percentage of Yes (2) ratings for every content area except for I. The size of the percentage changes in Yes (3) ratings was modest for every content area except H, where the increase was 20 percent.

Panelist-Level Results

Data for round 1 are graphed by panelist in figure 4. The panelist with the largest percentage of No ratings is Panelist 2 with 34 percent; the panelist with the smallest percentage is Panelist 4 with 4 percent. The panelist with the highest percentage of Yes (3) ratings is Panelist 5 with 48 percent; the panelist with the lowest percentage is Panelist 1 with 23 percent.

Round 2 data by panelist are graphed in figure 5. The panelist with the largest percentage of No ratings is now Panelist 1 at 20 percent. Panelists 3 and 4 have the lowest percentage of No ratings at 10 percent. Panelist 5 remains the panelist with the highest percentage of Yes (3) ratings, which increased from 48 percent to 52 percent. Panelist 1 continues to have the lowest percentage of Yes (3) ratings, increasing from 23 percent to 25 percent.

The differences across rounds at the panelist level can also be seen in figure 6. For Panelists 1, 2, and 5, the percentage of No ratings decreased, while the percentages of Yes ratings increased (though the pattern of the relative size of the increases for the different Yes levels varied across these panelists). Panelist 4 showed an increase in No ratings of 6 percent, and a decrease in Yes (1) ratings of 6 percent. For Panelist 3, an increase was seen for No and all levels of Yes ratings. This pattern reflects the fact that Panelist 3 had 22 descriptors for which no ratings were provided in round 1; therefore, the percentage increases also reflect added ratings for round 1, not just revised ratings.

Information about panelist changes is also given in table 9 and graphed in figure 7. Because of the 49 missing ratings in round 1 (see table 6), the data in table 9 provide a clearer picture of the number and types of changes made by each panelist. A total of 135 changes were made, with the largest number being made by Panelist 1 with 40, and the smallest number by Panelist 4 with 8.

For Panelist 1, all directional changes (not counting ratings that were blank in round 1 and provided in round 2) were made from No to Yes, or as increases in the level of Yes ratings; no ratings were changed from Yes to No or as decreases in the level of Yes ratings. Panelist 2 had changes from No to Yes and increases in the level of Yes ratings, but also had several decreases in the level of Yes ratings. Panelist 3 had no changes from No to Yes, or from Yes to No, but did have increases in the level of Yes ratings. Panelist 4 only had changes from Yes (1) to No. Panelist 5 had all but one of the changes from No to Yes, with the remaining change an increase in the level of a Yes rating. It should be noted that Panelist 5 was the only panelist who is a member of both the SAT and NAEP committees. As indicated by the response to the open-ended question on the final evaluation form (see Table 16), this panelist believed that more time should have been allowed for the review of the descriptors since “in mathematics, the language from document to document is seldom easily connected.” This panelist also indicated probably being able to assign items (rather than objectives) to descriptors. Thus it appears that in round 1, Panelist 5 was more stringent in the ratings of comparability given, and upon review of round 1 ratings from other panelists, saw a greater level of comparability and adjusted these ratings accordingly.

Overall Ratings

Summary-Level Results

Data on overall ratings are presented in table 10. All panelists agreed or strongly agreed that the content and breadth of the SAT and NAEP descriptors were comparable. All panelists indicated that there was sufficient overall overlap between the SAT and NAEP to justify conducting a more extensive alignment study at the item level.

Panelist-Level Results

Table 11 shows the overall ratings given by each of the panelists. Panelist 2 was the only panelist to profess strong agreement with a statement, specifically that the two tests covered the same content. All other ratings provided by panelists indicated that they agreed with the statements.

Panelist Feedback

Initial Evaluation

Panelist responses to the Initial Evaluation/Ready-to-Proceed form are shown in table 12. Four panelists strongly agreed and one panelist agreed that they understood the purpose of the study, and that the steps that they were to follow to make their content-area ratings were presented clearly. All panelists strongly agreed that the overview of the assessments was presented clearly. Two panelists strongly agreed and three panelists agreed that they understood what they would be expected to do to complete their content-area ratings. Table 13 shows responses by panelist.

In response to the last question, all panelists indicated that they were ready to proceed with round 1 ratings.

Final Evaluation

Panelist responses to the final evaluation form are shown in table 14. There were four sections to the form. In the first section, a question asked whether the SAT and NAEP skill statements were sufficiently detailed to judge comparability. All panelists agreed with this statement.

In the second section, panelists were asked four questions in relation to the content-area comparability ratings. They were asked (a) whether the content-area rating form was easy to complete, (b) whether the summary of the content-area ratings was presented clearly, (c) whether the discussion of the summary of content-area ratings was informative, and (d) whether the process of completing the content-area ratings was easy to follow. All panelists strongly agreed or agreed that the process of completing the content-area ratings was easy to follow (d). For the other three questions, however, panelists' ratings ranged from strongly agree to strongly disagree.

In the third section, the same questions asked in the previous section were repeated in relation to the overall ratings. Four panelists strongly agreed or agreed that the overall rating form was easy to complete; one panelist did not answer the question. Four panelists also strongly agreed or agreed that the summary of the overall ratings was presented clearly; one panelist replied "N/A, not enough time." All panelists strongly agreed or agreed that the process of completing the overall ratings was easy to follow. For the question asking about the summary of the overall ratings, one panelist each responded strongly agree, agree, disagree, and strongly disagree; again, one panelist replied "N/A, not enough time."

Ratings by panelist are shown in table 15. Panelist 1 answered "agree" to all questions. Panelist 2 answered "strongly agree" to "agree" to all but two questions, to which the panelist responded "N/A, not enough time." Panelist 3 had all but one response as "strongly agree," with the remaining response as "agree." Panelist 4 responded "agree" to five questions, "disagree" to three questions, and did not answer one question. Panelist 5 had four "strongly agree," two "agree," one "disagree," and two "strongly disagree" responses.

In the final section, panelists were asked what those running the study should consider doing differently the next time this type of study is conducted. The responses are shown in table 16. Many of the comments related to the fact that not enough time was allowed for the rating activities. Other comments concerned the differences in the level of specificity of the SAT and NAEP descriptors and the challenges that presented.

Discussion and Conclusions

Based on the results summarized above, the comparability study provided valuable information for the College Board and National Assessment Governing Board to consider when determining whether to proceed with an item-level alignment study. All panelists believed that such a study would be worthwhile given the level of comparability between the SAT performance characteristic descriptors and the NAEP objectives.

However, it is clear that the panelists felt rushed while providing their round 1 ratings, and would have preferred more time at the meeting to allow for all round 1 ratings to be

completed, feedback and discussion to take place, and round 2 ratings to then be provided. Their responses to the questions on the final evaluation form reflected these concerns, as did their comments in the open-ended section at the end of the form.

Given the challenge of providing round 2 ratings at home, the panelists took the task seriously, with the number of changes made ranging from 8 to 40 per panelist between round 1 and round 2. Although the greatest number of changes was made in the direction of No to Yes, panelists also adjusted the levels of their Yes ratings. In addition, one panelist had all changes moving from Yes to No. Thus, it is clear that they did take the time to compare their responses to those of other panelists. Although there were still some descriptors for which three panelists did not provide ratings during round 2, the number of missing ratings decreased sizably across rounds.

In summary, despite logistical challenges, the study yielded panelist feedback that the content and breadth of the SAT and NAEP descriptors were comparable, and that there is sufficient overall overlap between the SAT and NAEP to justify conducting a more extensive alignment study at the item level.

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Table 1

Panelist Characteristics

Panelist Number	Committee	Occupational Setting	Region*
1	SAT	K-12	Midwest
2	SAT	post-secondary	West
3	NAEP	K-12	South
4	SAT	K-12	South
5	NAEP, SAT	K-12	South

*Region as defined by the U.S. Census.

Table 2

Comparison of Panelist and Committee Characteristics

Characteristic	Committee		
	Comparability Study Panel	NAEP Standing Committee	SAT Standing Committee
Occupational Setting			
K-12	4 (80%)	12 (80%)	3 (30%)
Post-Secondary	1 (20%)	3 (20%)	7 (70%)
Region*			
Northeast	0 (0%)	3 (21%)	1 (10%)
South	3 (60%)	6 (43%)	4 (40%)
Midwest	1 (20%)	4 (29%)	1 (10%)
West	1 (20%)	1 (7%)	4 (40%)
Other (Puerto Rico)	0 (0%)	1 (7%)	0 (0%)

*Region as defined by the U.S. Census.

Table 3

*Number of Performance Characteristics Descriptors
for SAT Mathematics Section*

Content Area	Number of Descriptors
A. Numbers and Operations	17
B. Algebra and Functions	35
C. Geometry and Measurement	12
D. Data, Statistics, and Probability	9
E. Problem Solving	16
F. Representation	14
G. Reasoning	10
H. Connections	6
I. Communication	6
Total	125

Table 4

*Number of Objectives for NAEP 12th Grade
Mathematics Assessment*

Content Area	Number of Objectives
A. Number Properties and Operations	20
B. Measurement	18
C. Geometry	30
D. Data Analysis, Statistics, and Probability	32
E. Algebra	30
Total	130

Table 5

Meeting Agenda

8:00 a.m. – 8:30 a.m.	Continental Breakfast	
8:30 a.m. – 9:00 a.m.	Welcome Introduction Purpose of the Study	<i>Susan Loomis</i>
9:00 a.m. – 9:45 a.m.	Overview of the Assessments SAT Mathematics Grade 12 NAEP Mathematics	<i>David Banach</i> <i>David Garber</i>
9:45 a.m. – 10:00 a.m.	Panel Ratings Procedures for Study	<i>Mary Pitoniak</i>
10:00 a.m. – Noon	Panelists Reviews	
Noon – 1:00 p.m.	Lunch	
1:00 p.m. – 2:00 p.m.	Summarize Ratings & Discuss Summarize Ratings Discussion	<i>Mary Pitoniak</i> <i>Panels</i>
2:00 p.m. – 2:30 p.m.	Revisit Ratings	
2:30 p.m. – 3:00 p.m.	Wrap-up and Discussion	
3:00 p.m.	Adjourn	

Table 6

Ratings for Round 1

Content Area	Number of Descriptors	Number of Ratings*	Type of Rating				
			No	Yes (1)	Yes (2)	Yes (3)	Blank
A. Numbers and Operations	17	85	8 (9%)	14 (16%)	32 (38%)	29 (34%)	2 (2%)
B. Algebra and Functions	35	175	38 (22%)	36 (21%)	33 (19%)	64 (37%)	4 (2%)
C. Geometry and Measurement	12	60	10 (17%)	9 (15%)	18 (30%)	20 (33%)	3 (5%)
D. Data, Statistics, and Probability	9	45	4 (9%)	4 (9%)	7 (16%)	19 (42%)	11 (24%)
E. Problem Solving	16	80	24 (30%)	18 (23%)	16 (20%)	10 (13%)	12 (15%)
F. Representation	14	70	7 (10%)	8 (11%)	24 (34%)	30 (43%)	1 (1%)
G. Reasoning	10	50	17 (34%)	10 (20%)	7 (14%)	8 (16%)	8 (16%)
H. Connections	6	30	11 (37%)	6 (20%)	2 (7%)	6 (20%)	5 (17%)
I. Communication	6	30	6 (20%)	5 (17%)	9 (30%)	7 (23%)	3 (10%)
Total	125	625	125 (20%)	110 (18%)	148 (24%)	193 (31%)	49 (8%)

*There were 5 ratings (one for each panelist) for each descriptor. The numbers and percentages shown for each round are based on ratings, not descriptors.

Note. Yes (1) indicates weak comparability, Yes (2) indicates moderate comparability, and Yes (3) indicates strong comparability.

Table 7

Ratings for Round 2

Content Area	Number of Descriptors	Number of Ratings*	Type of Rating				
			No	Yes (1)	Yes (2)	Yes (3)	Blank
A. Numbers and Operations	17	85	3 (4%)	10 (12%)	39 (46%)	32 (38%)	1 (1%)
B. Algebra and Functions	35	175	33 (19%)	33 (19%)	41 (23%)	66 (38%)	2 (1%)
C. Geometry and Measurement	12	60	5 (8%)	9 (15%)	24 (40%)	21 (35%)	1 (2%)
D. Data, Statistics, and Probability	9	45	3 (7%)	8 (18%)	10 (22%)	23 (51%)	1 (2%)
E. Problem Solving	16	80	21 (26%)	22 (28%)	25 (31%)	10 (13%)	2 (3%)
F. Representation	14	70	2 (3%)	10 (14%)	26 (37%)	31 (44%)	1 (1%)
G. Reasoning	10	50	14 (28%)	17 (34%)	11 (22%)	8 (16%)	0 (0%)
H. Connections	6	30	7 (23%)	7 (23%)	4 (13%)	12 (40%)	0 (0%)
I. Communication	6	30	3 (10%)	9 (30%)	7 (23%)	9 (30%)	2 (7%)
Total	125	625	91 (15%)	125 (20%)	187 (30%)	212 (34%)	10 (2%)

*There were 5 ratings (one for each panelist) for each descriptor. The numbers and percentages shown for each round are based on ratings, not descriptors.

Note. Yes (1) indicates weak comparability, Yes (2) indicates moderate comparability, and Yes (3) indicates strong comparability.

Table 8

Number of Ratings Left Blank for Each Round by Panelist

Panelist	Round	
	1	2
1	8	0
2	6	6
3	22	0
4	2	2
5	11	2
Total	49	10

Note. The total number of ratings to be made in each round was 625.

Table 9

Number of Changes from Round 1 to Round 2 by Panelist

	Direction of Change	Panelist					Total
		1	2	3	4	5	
No to Yes	No to Yes (1)	12	10			13	35
	No to Yes (2)	2	4			3	9
	No to Yes (3)		3				3
Yes to No	Yes (1) to No				8		8
	Yes (2) to No						0
	Yes (3) to No						0
Increase in level of Yes rating	Yes (1) to Yes (2)	14	4	2		1	21
	Yes (1) to Yes (3)		1	3			4
	Yes (2) to Yes (3)	2	2	3			7
Decrease in level of Yes rating	Yes (2) to Yes (1)		2				2
	Yes (3) to Yes (2)		4				4
Blank to No	Blank to No			6			6
Blank to Yes	Blank to Yes (1)	3		6		2	11
	Blank to Yes (2)	5	1	6		2	14
	Blank to Yes (3)			4		5	9
Yes to Blank	Yes (1) to Blank	1					1
	Yes (3) to Blank	1					1
	Total	40	31	30	8	26	135

Note. The total number of ratings to be made in each round was 625.

Table 10

Summary of Overall Ratings

Topic	Rating			
	Strongly Agree	Agree	Disagree	Strongly Disagree
Content	1	4	0	0
Breadth	0	5	0	0
	Yes		No	
Future Study	5	0		

Table 11

Overall Ratings by Panelist

Topic	Panelist				
	1	2	3	4	5
Content	agree	strongly agree	agree	agree	agree
Breadth	agree	agree	agree	agree	agree
Future Study	yes	yes	yes	yes	yes

Table 13

Responses to Initial Evaluation Form by Panelist

Topic	Panelist				
	1	2	3	4	5
Understand purpose	strongly agree	strongly agree	strongly agree	agree	strongly agree
Overview presented clearly	strongly agree	strongly agree	strongly agree	strongly agree	strongly agree
Steps presented clearly	strongly agree	agree	strongly agree	strongly agree	strongly agree
Understand how to complete ratings	strongly agree	agree	strongly agree	agree	agree
Ready to Proceed	yes	yes	yes	yes	yes

Table 14

Summary of Responses to Final Evaluation Form

Statement	Rating				
	Strongly Agree	Agree	Disagree	Strongly Disagree	No or Other Response
The SAT and NAEP skill statements were sufficiently detailed to judge comparability.	0	5	0	0	0
<i>Content-Area Comparability Ratings</i>					
The content-area rating form was easy to complete.	2	1	2	0	0
The summary of our content-area ratings was presented clearly.	3	1	1	0	0
The discussion of the summary of content-area ratings was informative.	1	3	0	1	0
The process of completing the content-area ratings was easy to follow.	3	2	0	0	0
<i>Overall Ratings</i>					
The overall rating form was easy to complete.*	2	2	0	0	1
The summary of our overall ratings was presented clearly.*	2	2	0	0	1
The discussion of the summary of overall ratings was informative.*	1	1	1	1	1
The process of completing the overall ratings was easy to follow.	3	2	0	0	

*One panelist provided a rating of "N/A (not enough time)" for this question.

Table 15

Responses to Final Evaluation Form by Panelist

Area	Topic	Panelist				
		1	2	3	4	5
General	Descriptors detailed	agree	agree	agree	agree	agree
Content-Area Ratings	Form completion	agree	strongly agree	strongly agree	disagree	disagree
	Ratings summary	agree	strongly agree	strongly agree	disagree	strongly agree
	Discussion informative	agree	agree	strongly agree	agree	strongly disagree
	Process easy to follow	agree	strongly agree	strongly agree	agree	strongly agree
Overall Ratings	Form completion	agree	strongly agree	strongly agree		agree
	Ratings summary	agree	N/A (not enough time)	strongly agree	agree	strongly agree
	Discussion informative	agree	N/A (not enough time)	strongly agree	disagree	strongly disagree
	Process easy to follow	agree	strongly agree	strongly agree	agree	strongly agree

Table 16

Responses to Open-Ended Question on Final Evaluation Form by Panelist

Panelist	Response
1	The forms should be sent a couple of weeks in advance and the first round should be completed individually before the meeting. Then substantive discussions can occur during the in-person meeting.
2	I hope a better term than "comparability" can be found and used. Our discussion made clear the intent was to determine whether SAT item descriptors fell within the intent of the NAEP framework descriptors. It was a little like comparing apples and oranges since we had to work with Framework descriptors for NAEP and item descriptors for SAT. It might have been more effective to use the College Board Standards for Student Success that are to be addressed by SAT if that list has been prepared. At least this would be more like comparing apples and apples. Alternatively, once the new NAEP is written and item descriptors are available, it would probably be still better to compare NAEP item descriptors with SAT item descriptors. Probably the most accurate comparison would be using items from both tests. Such a comparison would be far more efficient if item descriptors for the items were also available and linked to their corresponding items. So, panelists could first match descriptors (in the intended direction) and then look at items to verify or correct their ratings.
3	The only thing that comes to mind is allowing more time for discussion. If we had two days to complete the task, we might have had valuable discussion to inform the ratings. Overall, it was highly organized and very pleasant! Thank you for all of your excellent preparation.
4	<i>For the rating of "disagree" for the item about the content-area rating form being easy to complete:</i> Performance vs. more global objectives made the comparison quite difficult at times. <i>For the rating of "disagree" for the item about the summary of the content-area ratings being presented clearly:</i> We didn't have time to summarize. <i>For the rating of "agree" for the item about the content-area ratings discussion being informative:</i> I wish we had discussed more. <i>For the rating of "disagree" for the item about the overall ratings discussion being informative:</i> There wasn't enough discussion. <i>Overall comments:</i> Since mathematicians read every word, we need more time to compare the two tests and then more time to discuss our ratings to determine whether or not we should imply a correlation or insist that the words be more exact. I would have preferred to have an introductory meeting Sunday evening so we could understand our task and perhaps get started on the comparison. Then we could actually "sleep on it" and decide if the instructions were clear and have more time on Monday to actually complete all the forms and discuss our differences. Thank you for soliciting our opinion.

(continued on next page)

Table 16 (continued)

Panelist	Response
5	<p><i>For the rating of "strongly agree" for the item about the summary of the content-area ratings being presented clearly: By email. For the rating of "strongly disagree" for the item about the content-area ratings discussion being informative: N/A...Didn't have time. For the rating of "strongly agree" for the item about the summary of the overall ratings being presented clearly: By email. For the rating of "strongly disagree" for the item about the overall ratings discussion being informative: N/A...Didn't have time. Overall comments: I would definitely allow more time for mathematics folks to assimilate, ponder, the content descriptors. In mathematics, the language from document to document is seldom easily connected. In this situation, one of the documents would address specific details (i.e., SAT's use of "unfamiliar symbols," odd and even numbers, "non-routine problems") that the NAEP document didn't. Also, the SAT document often had long lists of expectations (i.e., H4, H5, H6) for which there were some items that clearly correlated with NAEP items and others that did not. It was not clear (to me, at least) as to how to fairly rate the comparability of such entries. By allowing more time, we would have been able to come to much closer agreement than we have...even with the email option you provided us. I'm sorry for us, but more for the sake of good feedback to you) that we couldn't get to that discussion level. We all wanted to give you the best feedback, but time didn't allow. I do think that many of the SAT test items that I have seen that assess the individual descriptors could/would find a home in the NAEP descriptors. However, the difference in "wording" and the difference in the "specificity" addressed didn't always make that clear enough to declare "comparability."</i></p>

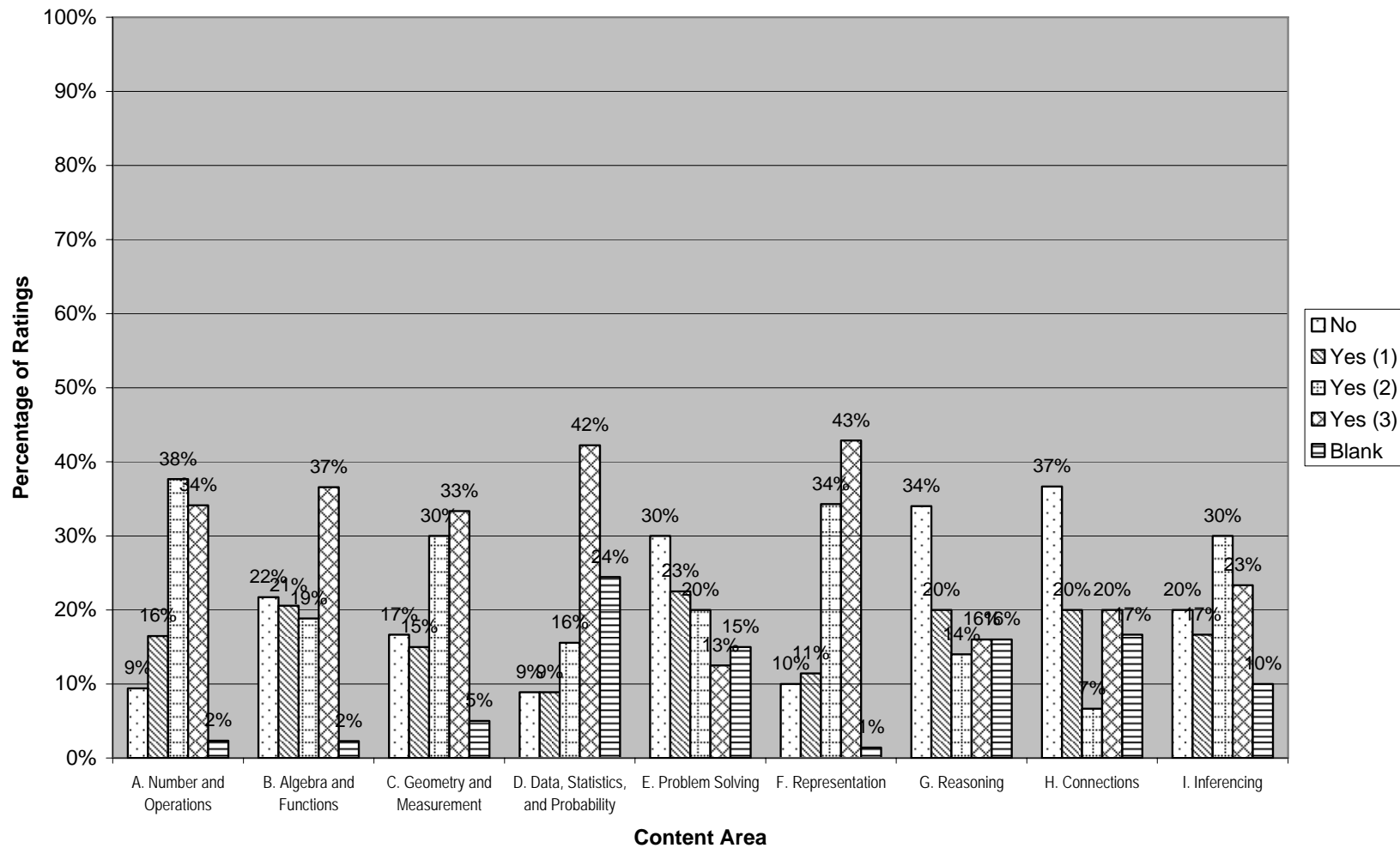


Figure 1. Round 1 Ratings by Content Area. For each content area, the percentage of ratings falling into each of the four rating categories is shown.

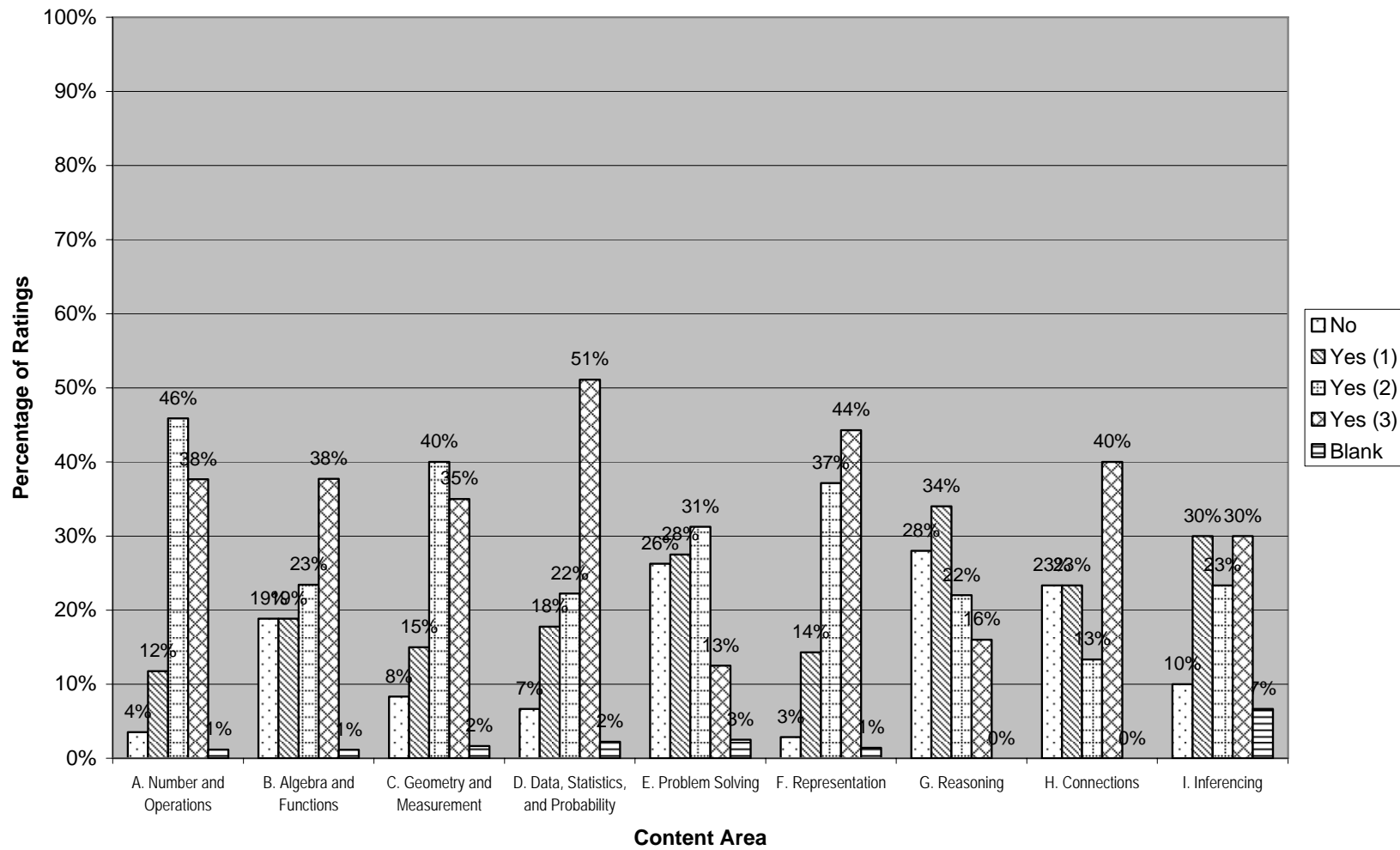


Figure 2. Round 2 Ratings by Content Area. For each content area, the percentage of ratings falling into each of the four rating categories is shown.

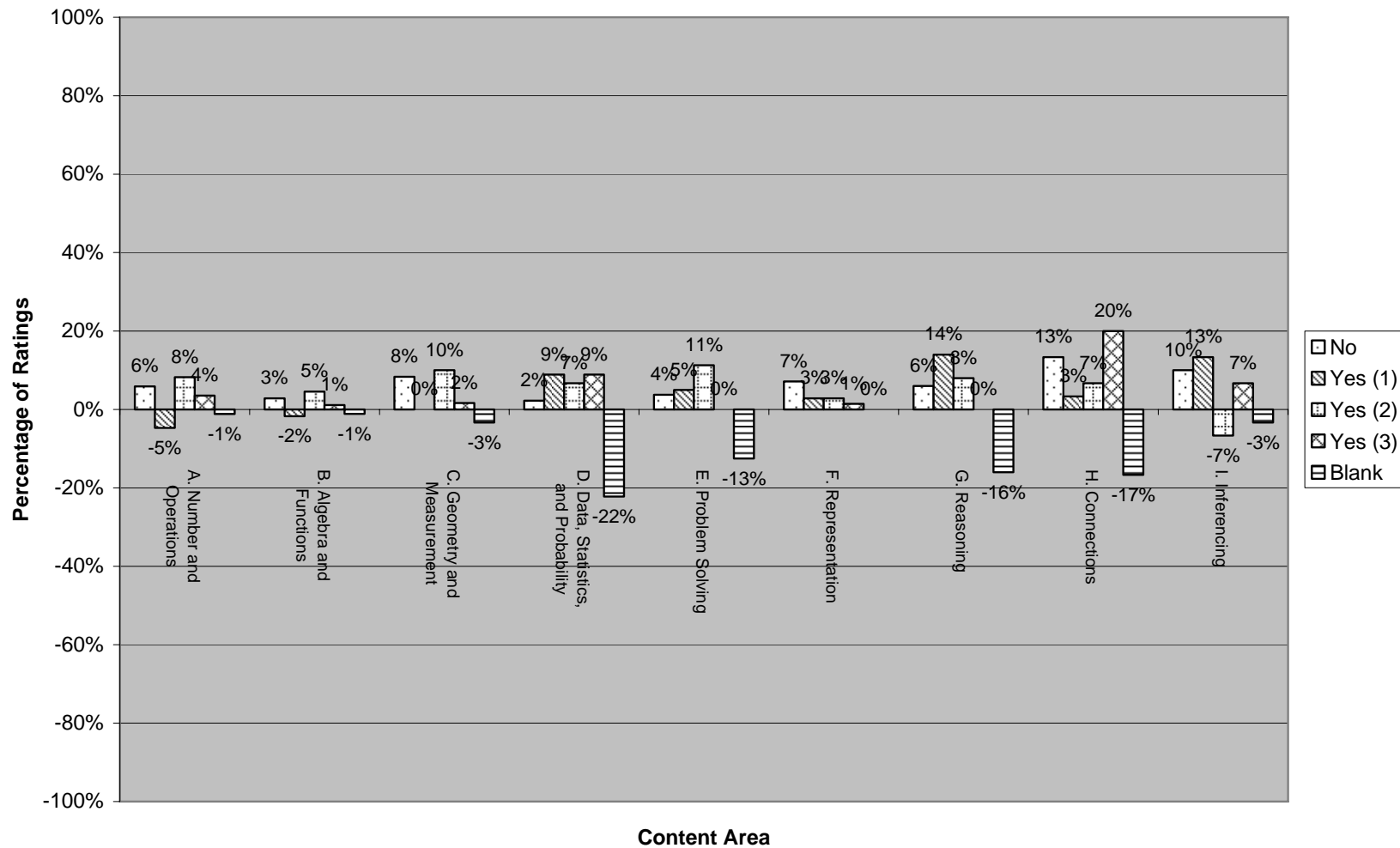


Figure 3. Changes in Ratings Over Rounds by Content Area. For each content area, the change in percentage of ratings falling into each of the four rating categories is shown.

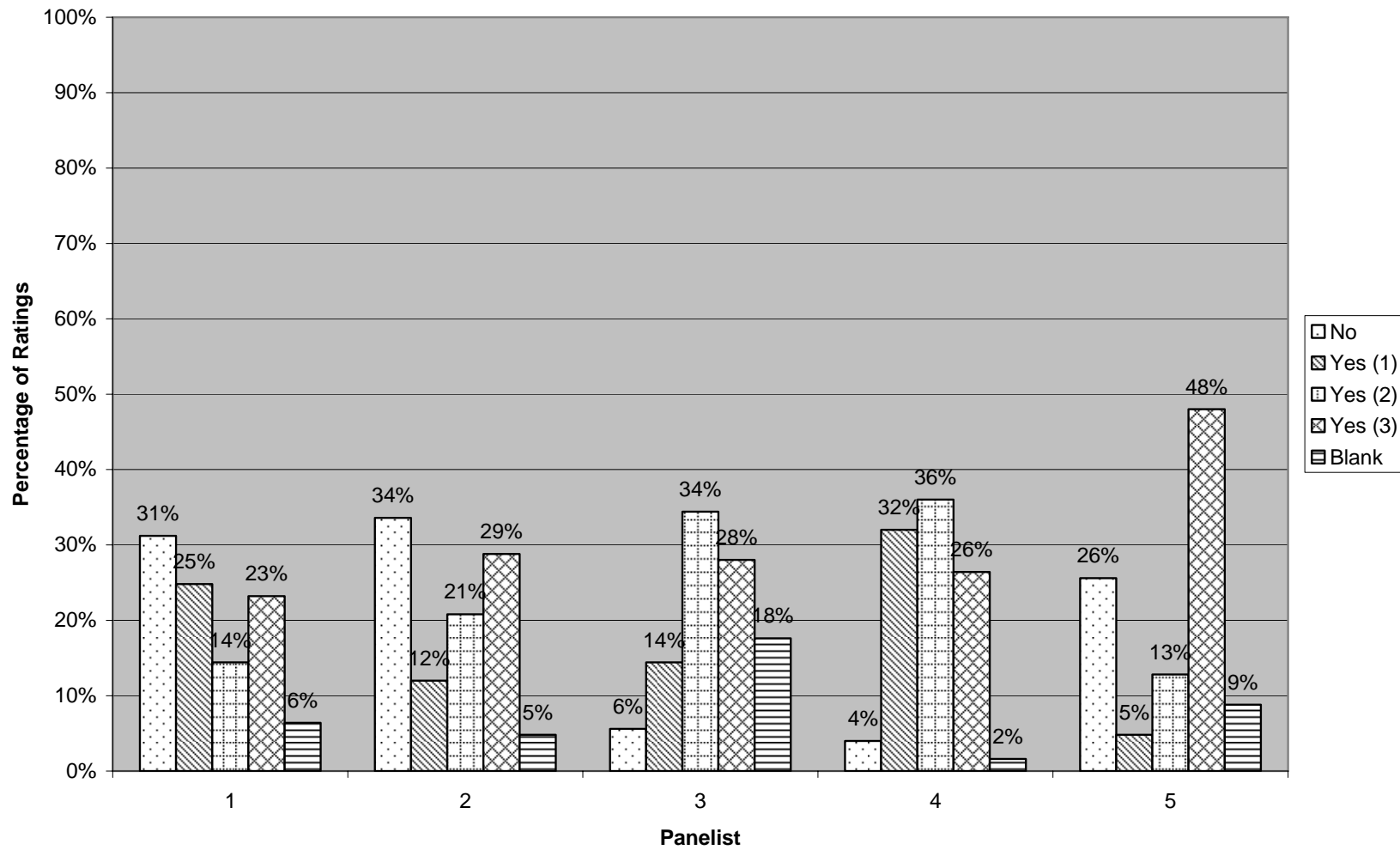


Figure 4. Round 1 Ratings by Panelist.
 For each panelist, the percentage of ratings falling into each of the four rating categories is shown.

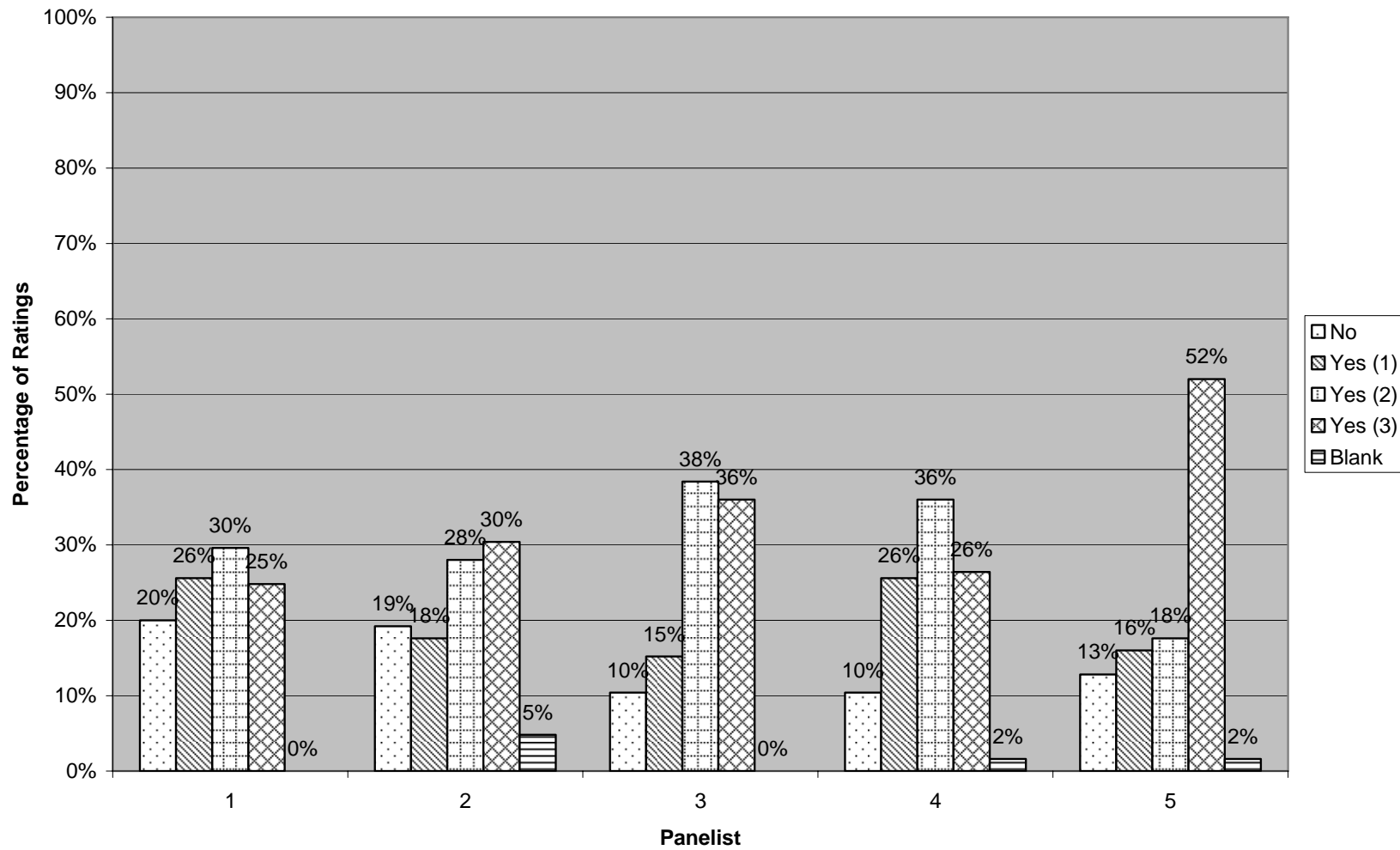


Figure 5. Round 2 Ratings by Panelist.
 For each panelist, the percentage of ratings falling into each of the four rating categories is shown.

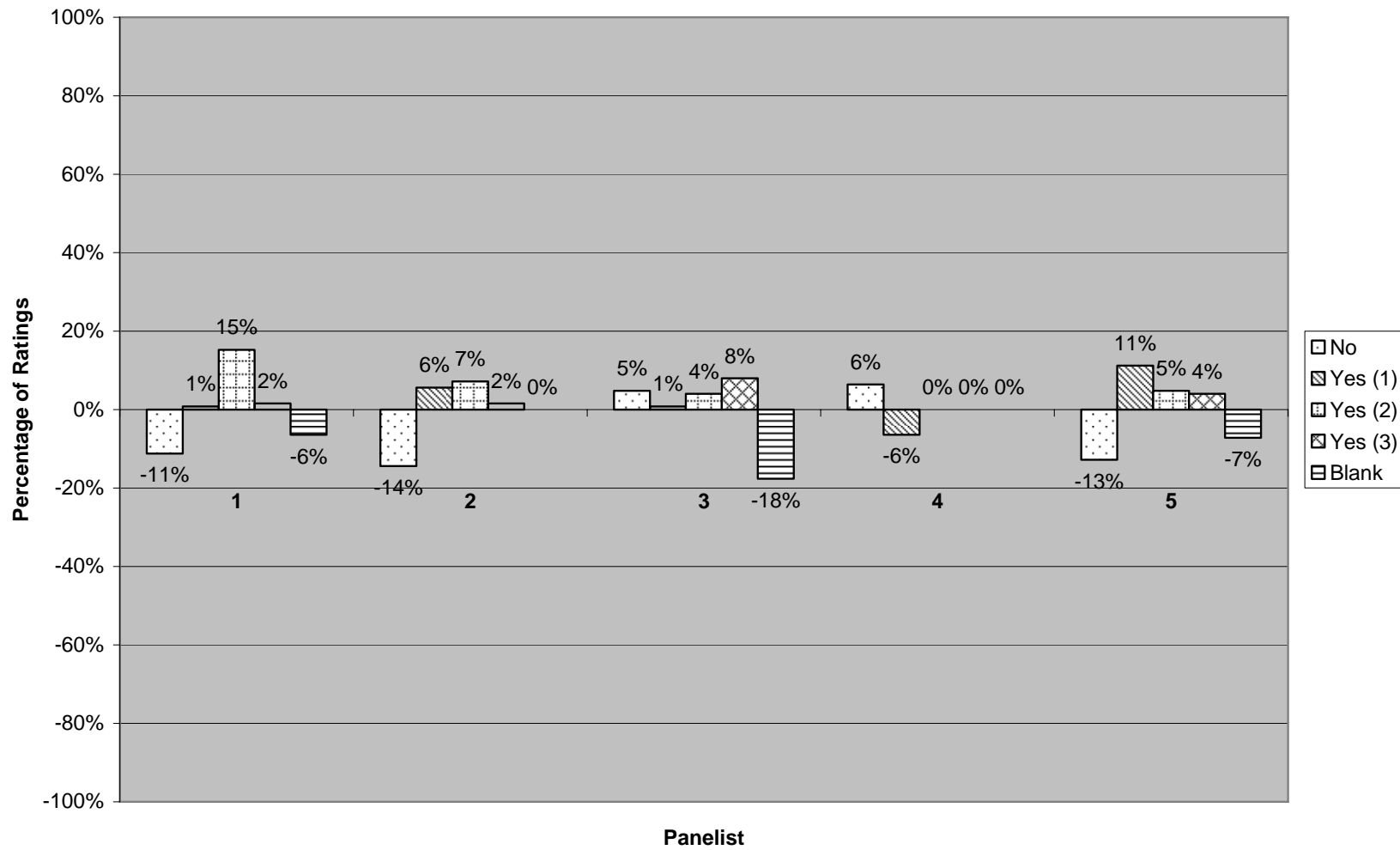


Figure 6. Changes in Ratings Over Rounds by Panelist. For each panelist, the change in percentage of ratings falling into each of the four rating categories is shown.

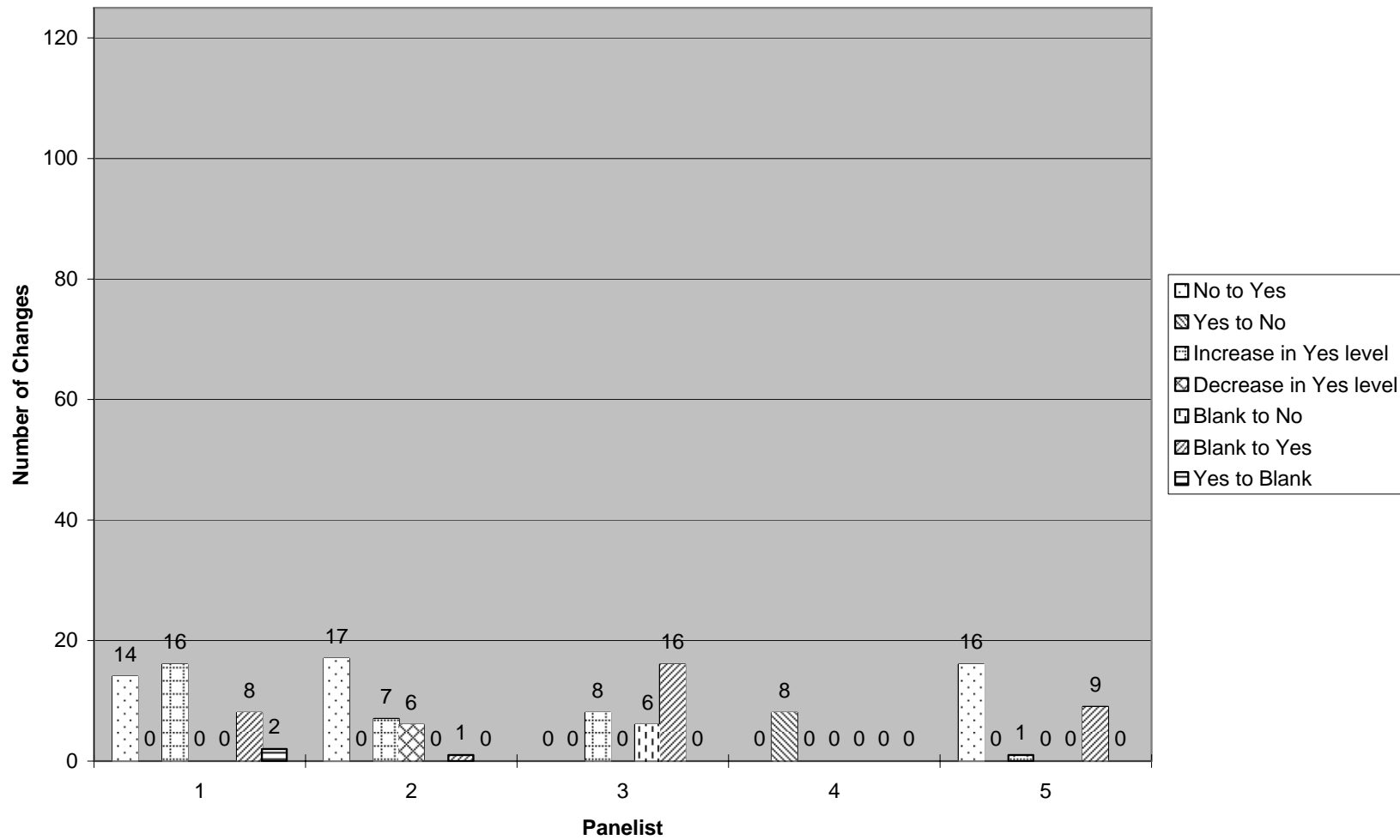


Figure 7. Types of Changes in Ratings Over Rounds by Panelist. For each panelist, the number of changes of each type is shown.

Appendix A

SAT Mathematics Performance Category Descriptors

A. Number and Operations

1. Identify factors of whole numbers
2. Solve word problems using addition, subtraction, multiplication, and division of whole numbers
3. Recall basic mathematical facts/definitions about exponential notation, including scientific notation
4. Identify a rule that describes a numerical pattern in a sequence
5. Identify, use, and represent fractions and percents in arithmetic and algebraic settings
6. Use properties of even and odd numbers, multiples, and factors
7. Identify and use the names for place values in solving problems involving decimal representations (e.g., tenths and hundredths)
8. Use properties of inequalities to compare and order numbers
9. Solve problems using ideas from basic set theory and basic number theory
10. Recognize and apply ratio, proportion, or percent in solving problems
11. Use properties of real number operations, ordering, and the zero-product property
12. Solve problems involving counting techniques
13. Determine values or properties of numbers in a sequence when given a description of the sequence
14. Create and use ratios, fractions, or percents in solving problems
15. Solve more-complex counting problems (e.g., permutations, combinations, and inclusion/exclusion)
16. Use π in algebraic and geometric contexts
17. Create and use ratios, fractions, or percents, including algebraic expressions, in solving problems

B. Algebra and Functions

1. Use letters as placeholders for unknown values
2. Treat expressions such as $a + b$ as a single quantity in linear problem situations (e.g., solving $2(a + b) = 6$ to find the value of $a + b$)
3. Verify that a value is a solution to a linear or quadratic equation (e.g., substitute and simplify)
4. Use function notation in simple situations (e.g., evaluation)
5. Use variables in multistep abstract settings (e.g., apply the distributive property across several variables)
6. Solve problems involving positive-integer exponents
7. Solve word problems involving linear relationships
8. Substitute values in and simplify systems of equations in two variables
9. Solve two-step linear equations
10. Evaluate an operation in two variables represented by unfamiliar symbols
11. Formulate and solve problems involving proportions
12. Solve multistep problems involving linear and quadratic relationships
13. Use and interpret graphs, including graphs of step functions
14. Solve problems involving algebraic inequalities
15. Solve problems involving exponential growth and decay
16. Evaluate an operation in three variables represented by unfamiliar symbols
17. Apply the concept of absolute value to algebraic expressions
18. Identify and analyze the qualitative behavior of graphs of nonlinear functions
19. Solve problems involving nonlinear functions and equations (e.g., quadratic, exponential, and rational)
20. Solve problems involving fractional and negative exponents
21. Identify solution sets in algebraic situations involving inequalities
22. Solve problems involving composition of functions (e.g., use the output of one function as the input to be evaluated in a second function)
23. Solve problems involving variables with operations represented by unfamiliar symbols
24. Generalize an exponential pattern from a geometric sequence
25. Solve for one variable or expression in terms of another
26. Work with systems of equations involving three or more variables
27. Solve problems involving complex fractions
28. Solve problems involving functions defined with unfamiliar symbols in one or more variables
29. Identify, apply, and represent transformations of functions, graphically and algebraically (e.g., vertical shift)
30. Apply properties of non-integer exponents
31. Solve multistep problems involving algebraic inequalities
32. Solve word problems involving rate of change in nonlinear or piecewise-linear settings
33. Identify and use the relationship between the slope of a line and algebraic rate of change
34. Interpret and solve word problems using multistep proportional reasoning
35. Transform an equation or expression by raising it to a power

C. Geometry and Measurement

1. Solve geometry problems involving basic shapes (e.g., triangles, circles, and segments)
2. Recall basic mathematical facts about triangles (e.g., properties of isosceles triangles and the 180° angle sum property)
3. Apply properties of triangles, including congruence
4. Apply angle relationships, including those in polygons and circles
5. Solve problems involving the length of line segments
6. Recognize and use the following:
 - Simple inscribed and circumscribed figures
 - The Pythagorean Theorem
 - Coordinate geometry (e.g., slope calculations)
 - Parallelism and perpendicularity
 - Two- and three-dimensional figures
 - Figures composed of two or more simple shapes
7. Interpret and solve two-step problems involving geometric proportions
8. Recognize and use volume in solving multistep problems
9. Use the relationships between the slopes of parallel and perpendicular lines in the coordinate plane
10. Determine the effect of changes in the linear dimensions of a figure on other measures of the figure, such as area
11. Interpret and solve multistep problems involving geometric proportions
12. Solve problems involving networks

D. Data, Statistics, and Probability

1. Read simple data displays (e.g., bar graphs, line graphs, pictograms, and tables)
2. Read and interpret bar graphs
3. Extract and use relevant information from tables, graphs, and diagrams
4. Interpret and solve problems involving data displays (e.g., circle graphs)
5. Interpret and solve multistep problems involving data displays
6. Interpret the effect of changes in data on measures of center
7. Solve problems involving probability
8. Solve conditional probability problems
9. Solve geometric probability problems

E. Problem Solving

1. Set up and solve one-step problems involving rates
2. Apply a simple procedure to solve an arithmetic problem
3. Solve one-step proportional reasoning problems
4. Read, extract, and use relevant information from written descriptions and geometric figures to solve a problem
5. Solve some multistep routine problems
6. Solve problems involving rates and unit conversions
7. Use multistep strategies to solve a problem, such as the following:
 - Drawing auxiliary lines
 - Breaking a larger problem down to smaller components
8. Solve multistep nonroutine problems (e.g., by trial and error)
9. Solve multistep geometry problems involving the following:
 - Angle measures and relationships
 - Triangles
10. Solve problems using multiple strategies, including the following:
 - Visualization
 - Estimation skills
 - Recognizing relevant information
 - Function notation
11. Use insight in solving nonroutine geometric problems involving the following:
 - Triangles
 - Patterns
 - Perimeter
 - The Pythagorean Theorem
 - Properties of circles
12. Solve the first stage of a problem, and then apply that solution to solve the next stage of the problem
13. Recognize complexity in problems that appear at first to be routine
14. Develop and apply an effective strategy and keep track of information in solving a nonroutine problem
15. Identify relevant and irrelevant information when choosing a solution strategy
16. Solve multistep problems involving properties of integers

F. Representation

1. Read pictorial and tabular representations to identify an answer
2. Select an appropriate representation for a proportion
3. Translate verbal statements into algebraic expressions
4. Create and apply an appropriate representation for a rate
5. Visualize or create a geometric representation to solve a problem
6. Translate between verbal and symbolic representations of linear expressions
7. Translate between equivalent symbolic representations of linear expressions
8. Recognize and translate among information represented verbally, graphically, numerically, and symbolically
9. Visualize or sketch a figure based on a verbal description to solve a problem
10. Interpret functions and graphs as models in applied situations
11. Translate verbal descriptions into algebraic representations in solving complex problems
12. Translate among equivalent representations of expressions involving exponents
13. Compare and contrast algebraic and geometric representations
14. Translate verbal descriptions into nonlinear algebraic representations in solving complex problems

G. Reasoning

1. Apply reasoning in solving straightforward problems in familiar settings
2. Reason about, structure, and solve problems about rates and proportions
3. Consider and compare different cases in reasoning about a problem situation
4. Make and test conjectures involving basic logic and set theory
5. Use basic number theory to investigate conjectures (e.g., conjectures about odd/even, positive/negative, and consecutive integers)
6. Recognize and use counterexamples
7. Consider multiple cases
8. Investigate and coordinate multiple conjectures to draw a logical conclusion
9. Decide which cases to consider in order to reach a conclusion
10. Make and test conjectures about properties of operations represented by unfamiliar symbols

H. Connections

1. Make connections between Data Analysis and Number and Operations (e.g., use numerical judgment in reading a simple data display)
2. Use variables in a geometric context (e.g., work with unknown angles identified by x and y)
3. Use variables in areas other than algebra
4. Use connections between areas of mathematics, such as the following:
 - Algebra and geometry (e.g., connect geometric slope with an algebraic expression)
 - Data and algebra (e.g., compute mean of algebraic expressions)
 - Applying proportions in geometric situations
5. Use connections between areas of mathematics, such as the following:
 - Coordinate Geometry and Algebra
 - Number and Operations and Data, Statistics, and Probability
 - Number and Operations and Geometry
 - Number and Operations and Algebra
 - Data, Statistics, and Probability and Geometry and Measurement
 - Algebra and Functions and Data, Statistics, and Probability
6. Solve nonroutine problems involving the application of concepts from the following:
 - Algebra and Functions and Number and Operations
 - Geometry and Measurement and Algebra and Functions
 - Data, Statistics, and Probability and Number and Operations

I. Communication

1. Use the following notation and terms:
 - Factor (whole number)
 - Radius
2. • Use the following notation and terms:
 - Congruent angles
3. Use the following notation and terms:
 - Function notation
 - Parallel
4. Use the following notation and terms:
 - Consecutive integers
 - “NOT,” “CANNOT,” “must,” “which of the following”
 - Arcs
 - Angle bisector
5. Use the following notation and terms:
 - Median
 - Random
6. Use the following notations and terms:
 - π
 - Tangent (line to a circle; circle to a circle)
 - “more than”
 - Symmetry about the origin

Appendix B

NAEP 12th Grade Mathematics Objectives

2009 NAEP Math Grade 12 Framework Objectives

A. Number Properties and Operations

1) Number sense

- d) Represent, interpret or compare expressions for real numbers, including expressions utilizing exponents and logarithms.
- f) Represent or interpret expressions involving very large or very small numbers in scientific notation.
- g) Represent, interpret or compare expressions or problem situations involving absolute values.
- i) Order or compare real numbers, including very large and very small real numbers.

2) Estimation

- b) Identify situations where estimation is appropriate, determine the needed degree of accuracy, and analyze* the effect of the estimation method on the accuracy of results.
- c) Verify solutions or determine the reasonableness of results in a variety of situations
- d) Estimate square or cube roots of numbers less than 1,000 between two whole numbers.

3) Number operations

- a) Find integral or simple fractional powers of real numbers.
- b) Perform arithmetic operations with real numbers, including common irrational numbers.
- c) Perform arithmetic operations with expressions involving absolute value.
- d) Describe the effect of multiplying and dividing by numbers including the effect of multiplying or dividing a real number by:
 - Zero, or
 - A number less than zero, or
 - A number between zero and one, or
 - One, or
 - A number greater than one
- f) Solve application problems involving numbers, including rational and common irrationals.

2009 NAEP Math Grade 12 Framework Objectives

A. Number Properties and Operations

4) Ratios and proportional reasoning

- c) Use proportions to solve problems (including rates of change).
- d) Solve multi-step problems involving percentages, including compound percentages.

5) Properties of number and operations

- c) Solve problems using factors, multiples, or prime factorization.
- d) Use divisibility or remainders in problem settings.
- e) Apply basic properties of operations, including conventions about the order of operations.
- f) Recognize properties of the number system—whole numbers, integers, rational numbers, real numbers, and complex numbers—recognize how they are related to each other, and identify examples of each type of number.

6) Mathematical reasoning using number

- a) Give a mathematical argument to establish the validity of a simple numerical property or relationship.
- b) Analyze or interpret a proof by mathematical induction of a simple numerical relationship.

2009 NAEP Math Grade 12 Framework Objectives

B. Measurement

1) Measuring physical attributes

- b) Determine the effect of proportions and scaling on length, areas and volume.
- c) Estimate, or compare perimeters or areas of two-dimensional geometric figures.
- d) Solve problems of angle measure, including those involving triangles or other polygons or parallel lines cut by a transversal.
- f) Solve problems involving perimeter or area of plane figures such as polygons, circles, or composite figures.
- h) Solve problems by determining, estimating, or comparing volumes or surface areas of three-dimensional figures.
- i) Solve problems involving rates such as speed, density, population density, or flow rates.

2) Systems of measurement

- a) Recognize that geometric measurements (length, area, perimeter, and volume) depend on the choice of a unit, and apply such units in expressions, equations, and problem solutions.
- b) Solve problems involving conversions within or between measurement systems, given the relationship between the units.
- d) Understand that numerical values associated with measurements of physical quantities are approximate, are subject to variation, and must be assigned units of measurement.
- e) Determine appropriate accuracy of measurement in problem situations (e.g., the accuracy of measurement of the dimensions to obtain a specified accuracy of area) and find the measure to that degree of accuracy.
- f) Construct or solve problems involving scale drawings.

2009 NAEP Math Grade 12 Framework Objectives

B. Measurement

3) Measurement in Triangles

- a) Solve problems involving indirect measurement.
- b) Solve problems using the fact that trigonometric ratios (sine, cosine, and tangent) stay constant in similar triangles.
- c) Use the definitions of sine, cosine, and tangent as ratios of sides in a right triangle to solve problems about length of sides and measure of angles.
- d) Interpret and use the identity $\sin^2\theta + \cos^2\theta = 1$ for angles θ between 0° and 90° ; recognize this identity as a special representation of the Pythagorean theorem.
- e) Determine the radian measure of an angle and explain how radian measurement is related to a circle of radius 1.
- f) Use trigonometric formulas such as addition and double angle formulas.
- g) Use the law of cosines and the law of sines to find unknown sides and angles of a triangle.

2009 NAEP Math Grade 12 Framework Objectives

C. Geometry

1) Dimension and shape

- c) Give precise mathematical descriptions or definitions of geometric shapes in the plane and in three-dimensional space.
- d) Draw or sketch from a written description plane figures and planar images of three-dimensional figures.
- e) Use two-dimensional representations of three-dimensional objects to visualize and solve problems.
- f) Analyze properties of three-dimensional figures including spheres and hemispheres.

2) Transformation of shapes and preservation of properties

- a) Recognize or identify types of symmetries (e.g., point, line, rotational, self-congruence) of two- and three-dimensional figures.
- b) Give or recognize the precise mathematical relationship (e.g., congruence, similarity, orientation) between a figure and its image under a transformation.
- c) Perform or describe the effect of a single transformation on two- and three-dimensional geometric shapes (reflections across lines of symmetry, rotations, translations, and dilations).
- d) Identify transformations, combinations or subdivisions of shapes that preserve the area of two-dimensional figures or the volume of three-dimensional figures.
- e) Justify relationships of congruence and similarity, and apply these relationships using scaling and proportional reasoning.
- g) Perform or describe the effects of successive transformations.

2009 NAEP Math Grade 12 Framework Objectives

C. Geometry

3) Relationships between geometric figures

- b) Apply geometric properties and relationships to solve problems in two and three dimensions.
- c) Represent problem situations with geometric models to solve mathematical or real world problems.
- d) Use the Pythagorean theorem to solve problems in two- or three-dimensional situations.
- e) Recall and interpret definitions and basic properties of congruent and similar triangles, circles, quadrilaterals, polygons, parallel, perpendicular and intersecting lines, and associated angle relationships.
- f) Analyze properties or relationships of triangles, quadrilaterals, and other polygonal plane figures.
- g) Analyze properties and relationships of parallel, perpendicular, or intersecting lines, including the angle relationships that arise in these cases.
- h) Analyze properties of circles and the intersections of lines and circles (inscribed angles, central angles, tangents, secants, chords).

4) Position, direction, and coordinate geometry

- a) Solve problems involving the coordinate plane such as the distance between two points, the midpoint of a segment, or slopes of perpendicular or parallel lines.
- b) Describe the intersections of lines in the plane and in space, intersections of a line and a plane, or of two planes in space.
- c) Describe or identify conic sections and other cross sections of solids.
- d) Represent two-dimensional figures algebraically using coordinates and/or equations.
- e) Use vectors to represent velocity and direction; multiply a vector by a scalar and add vectors both algebraically and graphically.
- f) Find an equation of a circle given its center and radius and, given an equation of a circle, find its center and radius.
- g) Graph ellipses and hyperbolas whose axes are parallel to the coordinate axes and demonstrate understanding of the relationship between their standard algebraic form and their graphical characteristics.
- h) Represent situations and solve problems involving polar coordinates.

2009 NAEP Math Grade 12 Framework Objectives

C. Geometry

5) Mathematical reasoning in Geometry

- a) Make, test, and validate geometric conjectures using a variety of methods including deductive reasoning and counterexamples.
- b) Determine the role of hypotheses, logical implications, and conclusion, in proofs of geometric theorems.
- c) Analyze or explain a geometric argument by contradiction
- d) Analyze or explain a geometric proof of the Pythagorean theorem.
- e) Prove basic theorems about congruent and similar triangles and circles.

2009 NAEP Math Grade 12 Framework Objectives

D. DATA ANALYSIS, STATISTICS, AND PROBABILITY

1) Data representation

The following representations of data are indicated for grade 12: Histograms, line graphs, scatterplots, box plots, bar graphs, circle graphs, stem and leaf plots, frequency distributions, and tables, including two-way tables.

Objectives in which only a subset of these representations is applicable are indicated in the parenthesis associated with the objective.

- a) Read or interpret graphical or tabular representations of data.
- b) For a given set of data, complete a graph and solve a problem using the data in the graph (histograms, scatterplots, line graphs)
- c) Solve problems involving univariate or bivariate data.
- d) Given a graphical or tabular representation of a set of data, determine whether information is represented effectively and appropriately.
- e) Compare and contrast different graphical representations of univariate and bivariate data.
- f) Organize and display data in a spreadsheet in order to recognize patterns and solve problems.

2) Characteristics of data sets

- a) Calculate, interpret, or use summary statistics for distributions of data including measures of typical value (mean, median), position (quartiles, percentiles), and spread (range, interquartile range, variance, standard deviation).
- b) Recognize how linear transformations of one-variable data affect mean, median, mode, range, interquartile range, and standard deviation.
- c) Determine the effect of outliers on mean, median, mode, range, interquartile range, or standard deviation.
- d) Compare data sets using summary statistics (mean, median, mode, range, interquartile range, or standard deviation) describing the same characteristic for two different populations or subsets of the same population.
- e) Approximate a trend line if a linear pattern is apparent in a scatterplot or use a graphing calculator to determine a least-squares regression line, and use the line or equation to make predictions.
- f) Recognize that the correlation coefficient is a number from -1 to $+1$ that measures the strength of the linear relationship between two variables; visually estimate the correlation coefficient (e.g., positive or negative, closer to 0, .5, or 1.0) of a scatterplot.
- g) Know and interpret the key characteristics of a normal distribution such as shape, center (mean), and spread (standard deviation).

2009 NAEP Math Grade 12 Framework Objectives

D. DATA ANALYSIS, STATISTICS, AND PROBABILITY

3) Experiments and samples

- a) Identify possible sources of bias in sample surveys, and describe how such bias can be controlled and reduced.
- b) Recognize and describe a method to select a simple random sample.
- c) Draw inferences from samples, such as estimates of proportions in a population, estimates of population means, or decisions about differences in means for two "treatments".
- d) Identify or evaluate the characteristics of a good survey or of a well-designed experiment.
- e) Recognize the differences in design and in conclusions between randomized experiments and observational studies.

4) Probability

- a) Recognize whether two events are independent or dependent.
- b) Determine the theoretical probability of simple and compound events in familiar or unfamiliar contexts.
- c) Given the results of an experiment or simulation, estimate the probability of simple or compound events in familiar or unfamiliar contexts.
- d) Use theoretical probability to evaluate or predict experimental outcomes.
- e) Determine the number of ways an event can occur using tree diagrams, formulas for combinations and permutations, or other counting techniques.
- h) Determine the probability of independent and dependent events.
- i) Determine conditional probability using two-way tables.
- j) Interpret and apply probability concepts to practical situations.
- k) Use the binomial theorem to solve problems.

2009 NAEP Math Grade 12 Framework Objectives

D. DATA ANALYSIS, STATISTICS, AND PROBABILITY

5) Mathematical Reasoning With Data

- a) Identify misleading uses of data in real-world settings and critique different ways of presenting and using information.
- b) Distinguish relevant from irrelevant information, identify missing information, and either find what is needed or make appropriate approximations.
- c) Recognize, use, and distinguish between the processes of mathematical (deterministic) and statistical modeling.
- d) Recognize when arguments based on data confuse correlation with causation.
- e) Recognize and explain the potential errors caused by extrapolating from data.

2009 NAEP Math Grade 12 Framework Objectives

E. ALGEBRA

1) Patterns, relations, and functions

- a) Recognize, describe, or extend numerical patterns, including arithmetic and geometric progressions.
- b) Express linear and exponential functions in recursive and explicit form given a table, verbal description, or some terms of a sequence.
- e) Identify or analyze distinguishing properties of linear, quadratic, reciprocal inverse proportionality ($y=k/x$), exponential, or trigonometric functions from tables, graphs, or equations.
- g) Determine whether a relation, given in verbal, symbolic, tabular, or graphical form, is a function.
- h) Recognize and analyze the general forms of linear, quadratic, inverse proportionality ($y=k/x$), exponential, or trigonometric functions.
- i) Determine the domain and range of functions given in various forms and contexts.
- j) Given a function, determine its inverse if it exists, and explain the contextual meaning of the inverse for a given situation.

2) Algebraic representations

- a) Create and translate between different representations of algebraic expressions, equations, and inequalities (e.g., linear, quadratic, exponential, or trigonometric) using symbols, graphs, tables, diagrams, or written descriptions.
- b) Analyze or interpret relationships expressed in symbols, graphs, tables, diagrams (including Venn diagrams), or written descriptions and evaluate the relative advantages or disadvantages of different representations to answer specific questions.
- d) Perform or interpret transformations on the graphs of linear, quadratic, exponential, and trigonometric functions.
- e) Make inferences or predictions using an algebraic model of a situation.
- f) Given a real-world situation, determine if a linear, quadratic, inverse proportionality ($y=k/x$), exponential, logarithmic, or trigonometric function fits the situation.
- g) Solve problems involving exponential growth and decay.
- h) Analyze properties of exponential, logarithmic, and inverse proportionality ($y=k/x$) functions.

2009 NAEP Math Grade 12 Framework Objectives

E. ALGEBRA

3) Variables, expressions, and operations

- b) Write algebraic expressions, equations, or inequalities to represent a situation.
- c) Perform basic operations, using appropriate tools, on algebraic expressions including polynomial and rational expressions.
- d) Write equivalent forms of algebraic expressions, equations, or inequalities to represent and explain mathematical relationships.
- e) Evaluate algebraic expressions, including polynomials and rational expressions.
- f) Use function notation to evaluate a function at a specified point in its domain and combine functions by addition, subtraction, multiplication, division, and composition.
- g) Determine the sum of finite and infinite arithmetic and geometric series.
- h) Use basic properties of exponents and logarithms to solve problems.

4) Equations and inequalities

- a) Solve linear, rational or quadratic equations or inequalities, including those involving absolute value.
- c) Analyze situations, develop mathematical models, or solve problems using linear, quadratic, exponential, or logarithmic equations or inequalities symbolically or graphically.
- d) Solve (symbolically or graphically) a system of equations or inequalities and recognize the relationship between the analytical solution and graphical solution.
- e) Solve problems involving special formulas such as: $A = P(I + r)^t$, $A = Pe^{rt}$.
- f) Solve an equation or formula involving several variables for one variable in terms of the others.
- g) Solve quadratic equations with complex roots.

5) Mathematical Reasoning in Algebra

- a) Use algebraic properties to develop a valid mathematical argument.
- b) Determine the role of hypotheses, logical implications, and conclusions in algebraic argument.
- c) Explain the use of relational conjunctions (and, or) in algebraic arguments.

Appendix C

National Assessment Governing Board PowerPoint Presentation on Preparedness



Reporting Preparedness for Grade 12 NAEP

**Goals of the Governing Board for
Reading and Mathematics NAEP**

**Susan Cooper Loomis
Assistant Director, Psychometrics**



Overview

- Recommendation to the Board in 2004 by a national commission appointed to address issues related to 12th grade NAEP
- Board commissioned papers, appointed Ad Hoc Committees, and a Technical Panel on 12th Grade Preparedness Research to help address the goal of reporting preparedness for 12th grade NAEP



Members of the Technical Panel on 12th Grade Preparedness Research

Michael Kirst (Chair)

Stanford University

John Campbell

University of Minnesota

Mark David Milliron

Catalyze Learning International

David T. Conley

University of Oregon

Robert Mislevy

University of Maryland

Michael Kane

**National Conference of
Bar Examiners**

George C. Thornton, III

Colorado State University



Preparedness for Post-Secondary Activities

- ❑ Higher Education and Workplace (job or job training programs—civilian or military)
- ❑ Academic preparation, not behaviors that are known to be important indicators of readiness for college or workplace
- ❑ Preparedness means “remediation free;” eligible for placement in college credit-bearing course or job/job training program in reading/mathematics



Types of Prospective Studies

- Content alignment between NAEP and other assessments: a necessary first step
- Identifying NAEP scores and score ranges indicating preparedness via:
 - Judgments by subject matter experts
 - Statistical relationships with performance on other assessments



Content Alignment Studies

- ❑ Evaluate extent of content overlap between NAEP and other assessments
- ❑ Small-scale studies as a preliminary step
- ❑ Full-scale studies guide later statistical analyses



Purpose of this Study

- Get an early signal on the feasibility of using the SAT as an indicator of preparedness for NAEP
 - Want to develop statistical relationship to report SAT scores associated with preparedness in mathematics for placement in college credit courses and preparedness for workplace training program
 - Want to report percentage of students on NAEP that score at a level indicative of College Success, based on SAT data



Materials for Preliminary Alignment Study

□ Framework for NAEP Mathematics

- Detailed information on objectives within different mathematics content areas for NAEP

□ Statements of what students who score in specific score ranges of the mathematics SAT know and can do in mathematics



Questions?

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Appendix D

SAT Mathematics Content Overview PowerPoint Presentation



Overview of SAT Mathematics Reasoning Test

David Banach, ETS
February 4, 2008



What does the SAT do?

- Provides an objective measurement of students' readiness for college work
- Helps colleges make fair and informed decisions about applicants
- Measures critical thinking skills in 3 separate areas: Reading, Mathematics, and Writing
- Math sections measure quantitative reasoning skills that demonstrate how well students can analyze and solve problems

The SAT is a proven, reliable indicator of college success.



Information about the SAT Reasoning Test

- College-bound students typically take the SAT Reasoning Test during both their junior and senior years of high school.
- Test consists of 10 separately-timed sections. Total testing time is 3 hours 45 minutes.
- Each section contains either reading, writing, or mathematics questions.
- One section of the test contains questions that do not contribute to a student's score. This section is used to pretest questions that could become operational on future SAT forms.



What is the SAT Math Test like for students?

- There are a total of 54 questions: 44 multiple choice and 10 student-produced responses (SPR).
- SPR questions require students to solve a problem and grid a numerical value on their answer sheet.
- Math questions occur in three operational sections: M1, M2, and M3.
- M1 contains 20 multiple choice (25 min.)
- M2 contains 8 multiple choice and 10 SPR (25 min.)
- M3 contains 16 multiple choice (20 min.)



Calculator Policy

- It is recommended that students bring a calculator (either scientific or graphing) to use on the mathematics sections of the test.
- Every question can be solved without a calculator; however, using a calculator on some questions may be helpful.
- Calculators with computer algebra system (CAS) capabilities are allowed (e.g., TI-89).
- Calculators with QWERTY keypads or that are stylus-driven or that can communicate wirelessly with other devices are not allowed.



Mathematics Content on the SAT

- Number and Operations (20-25%)
- Algebra and Functions (35-40%)
- Geometry and Measurement (25-30%)
- Data Analysis, Statistics, and Probability (10-15%)

The questions deal with mathematics topics that college-bound students typically encounter during their first 3 years of high school (i.e., in Algebra I, Geometry, and Algebra II courses).



Number and Operations (20-25%)

- Arithmetic word problems (including percent, ratio, and proportion)
- Properties of integers (even, odd, prime numbers, divisibility, remainders, LCM, GCF, etc.)
- Rational numbers (including scientific notation and place value)
- Sets (union, intersection, elements, subsets, etc.)
- Counting techniques
- Sequences and series (including exponential growth)
- Elementary number theory



Algebra and Functions (35-40%)

- Substitution and simplifying algebraic expressions
- Properties of exponents (integer and rational)
- Algebraic representation and word problems
- Linear equations and inequalities
- Systems of equations and inequalities
- Quadratic equations
- Rational and radical equations
- Absolute value
- Direct and inverse variation
- Concepts of algebraic functions (symbolic, graphical, and tabular)
- Graphs of linear and quadratic functions
- Functions as models



Geometry and Measurement (25-30%)

- Area and perimeter of polygons
- Area and circumference of circles
- Volume and surface area of solids
- General properties of triangles
- Pythagorean theorem and special properties of isosceles, equilateral, and right triangles
- Properties of parallel and perpendicular lines
- Coordinate geometry (including concept of slope)
- Geometric visualization and perception
- Similarity and congruence
- Geometric transformations



Data Analysis, Statistics, and Probability (10-15%)

- Data interpretation (tables, charts, bar graphs, line graphs, circle graphs, histograms, pictographs, scatterplots)
- Descriptive statistics and measures of central tendency (e.g., mean, median, mode)
- Probability (elementary and geometric)



Reasoning Tests vs. Achievement Tests

- Few of the math questions on the SAT would be characterized as typical or routine textbook exercises that assess a single mathematical skill or algorithm.
- As a reasoning test, the content domain of the SAT is fairly modest when compared to achievement tests.
- Many questions require students to have a solid conceptual understanding of elementary mathematical principles and to apply these principles to novel problems.
- Questions are difficult not because they assess more advanced mathematical content (e.g., precalculus and beyond), but because they require synthesis and application of more elementary concepts.



Questions?

Comments?

Appendix E

NAEP 12th Grade Mathematics Content Overview PowerPoint Presentation



Overview of the Grade 12 NAEP Mathematics Assessment

Dave Garber, ETS
February 4, 2008



What is NAEP?

- Only nationally representative sample and continuing assessment of what America's students know and can do
- NAEP is the assessment used to write the Nation's Report Card
<http://nces.ed.gov/nationsreportcard/>
- Survey based on a representative sample of students; no individual scores are reported
- Goal is to measure student performance over time; results are scale scores and percentiles, with respect to achievement levels (basic, proficient, and advanced)
- Administered at the national, state, and selected urban district levels
- Assessments are given in the following subject areas: mathematics, reading, science, writing, civics, history, economics, arts, geography, foreign language



Overview of the NAEP Mathematics Assessment

- Administered at grades 4, 8, and 12
- Combination of multiple-choice and constructed-response questions
 - Assessment contains a mix of short response and extended response questions
- Five major content areas: Number properties & operations, Measurement, Geometry, Data analysis, statistics, & probability, and Algebra
- All NAEP Mathematics items are also classified with a level of complexity (low, moderate, or high), which is a measure of the cognitive demands an item makes on the student



What is NAEP Mathematics like for students?

- Students are given an assessment booklet for one subject only
- A booklet contains two 25-minute cognitive blocks
 - A “block” is a group of approximately 14-16 questions
 - Total testing time is about one hour for the student to take two cognitive blocks and answer some background questions
- The full grade 12 mathematics assessment will contain 12 cognitive blocks



Calculator Policy and Ancillary Materials Used on NAEP

- Calculator Policy
 - Grade 4: Students are provided with a 4-function calculator
 - Grades 8 & 12:
 - Students are allowed to bring their own calculator, including graphing calculators and those with CAS capabilities
 - Calculators with QWERTY keypads are not allowed
 - Students are provided with a scientific calculator if they don't bring their own

- Other ancillary materials (aka, “manipulatives”)
 - Spinners, number tiles, fraction strips, geometric shapes, rulers/protractors, etc.



Calculator Policy and Ancillary Materials Used on NAEP, Continued

- Students will have a calculator available to use for all the items in a calculator block, but not every item requires the use of a calculator
- All the items in a calculator block can be solved without a calculator; however, using a calculator on some items is often very helpful
- Care is taken during block assembly to ensure that no item in a calculator block provides an advantage to students with a graphing calculator or CAS-capable calculator
- A manipulative block contains a group of items (usually 4 or 5) that make use of the manipulative; the remaining questions in the block do not



2009 NAEP Mathematics Framework

- New objectives at grade 12; objectives at grades 4 and 8 did not change
- Revised to allow for reporting on how well 12th grade students are prepared for post-secondary education and training
- New reasoning subtopic has been added to 4 of the 5 content areas
- Objectives marked with an * indicate mathematical content beyond what is taught in a typical 3-year sequence of high school mathematics (1 year of geometry and 2 years of algebra)



5 NAEP Mathematics Content Areas

Table 1. Percentage Distribution of Items by Grade and Content Area

Content Area	Grade 4	Grade 8	Grade 12
Number Properties and Operations	40%	20%	10%
Measurement	20%	15%	30%
Geometry	15%	20%	
Data Analysis, Statistics, and Probability	10%	15%	25%
Algebra	15%	30%	35%



Sample Objectives

Algebra

4) Equations and inequalities		
GRADE 4	GRADE 8	GRADE 12
a) Find the value of the unknown in a whole number sentence.	a) Solve linear equations or inequalities (e.g., $ax + b = c$ or $ax + b = cx + d$ or $ax + b > c$).	a) Solve linear, rational or quadratic equations or inequalities, including those involving absolute value.
	b) Interpret "=" as an equivalence between two expressions and use this interpretation to solve problems.	
	c) Analyze situations or solve problems using linear equations and inequalities with rational coefficients symbolically or graphically (e.g., $ax + b = c$ or $ax + b = cx + d$).	c) Analyze situations, develop mathematical models, or solve problems using linear, quadratic, exponential, or logarithmic equations or inequalities symbolically or graphically.



Number Properties and Operations

Subtopics

1. Number sense
 2. Estimation
 3. Number operations
 4. Ratios and proportional reasoning
 5. Properties of number and operations
 6. Mathematical reasoning using number
- Emphasis at 12th grade is on real and complex numbers, as well as using numerical properties to provide/analyze mathematical arguments



Measurement and Geometry

Measurement Subtopics

1. Measuring physical attributes
2. Systems of measurement
3. Measurement in triangles

Geometry Subtopics

1. Dimension and shape
2. Transformation of shapes and preservations of properties
3. Relationships between geometric figures
4. Position, direction, and coordinate geometry
5. Mathematical reasoning in geometry

- Measurement and geometry are combined because many of the measurement topics by grade 12 are geometric in nature.
- Objectives assessing topics in trigonometry are included and students are expected to be familiar with analytical geometry techniques. Providing or analyzing geometric proofs has also been added to the 12th grade objectives.



Data Analysis, Statistics, and Probability

Subtopics

1. Data representation
 2. Characteristics of data sets
 3. Experiments and samples
 4. Probability
 5. Mathematical reasoning with data
- By 12th grade, students are expected to be able to use a variety of statistical techniques to evaluate designs of experiments, read and analyze various types of data, and formally solve problems involving probability



Algebra

Subtopics

1. Patterns, relations, and functions
 2. Algebraic representations
 3. Variables, expressions, and operations
 4. Equations and inequalities
 5. Mathematical reasoning in algebra
- By 12th grade, the concept of functions is addressed more formally and is expanded to include quadratic and some other non-linear functions



Sample Grade 12 Pilot Blocks

- **One non-calculator block**
- **One calculator block**



Questions?

Comments?

Appendix F

Rating Process Training PowerPoint Presentation



NAEP Grade 12 Mathematics and SAT Mathematics Assessments: Are They Comparable?

**Mary J. Pitoniak, ETS
February 4, 2008**



Overview of Tasks

1. Content-area ratings

- Yes/no
- Level of comparability

2. Overall ratings

- Content
- Breadth
- Advisability of future item-level study



What Do We Mean By Comparability?

- It refers to whether the content covered by the SAT Mathematics test is also covered by the NAEP grade 12 Mathematics test.
- So, for example, if SAT addresses a student's ability to *identify factors of whole numbers*, the question is: Does the NAEP test also address this ability?
- Comparability is based on your expert judgment:
 - Do you believe that what's on the SAT Mathematics test is also on the NAEP grade 12 Mathematics test?



What Do We Mean By Comparability? *(continued)*

- The question of comparability goes in one direction:
Does NAEP cover what's on the SAT?
- It does **not** ask:
Does SAT cover what's on the NAEP?



What Materials Will We Use To Judge Comparability?

- For **SAT**, statements of the Mathematics skills needed to answer the range of questions on the test—termed *performance characteristics descriptors*
- SAT descriptors are organized by 9 main categories:
 1. Numbers and Operations
 2. Algebra and Functions
 3. Geometry and Measurement
 4. Data, Statistics, and Probability
 5. Problem Solving
 6. Representation
 7. Reasoning
 8. Connections
 9. Communication



What Materials Will We Use To Judge Comparability? *(continued)*

- For **NAEP**, statements of the Mathematics skills that students at grade 12 should have—termed *objectives*
- NAEP objectives are organized by 5 areas of content:
 1. **Number Properties and Operations**
 2. **Measurement**
 3. **Geometry**
 4. **Data Analysis, Statistics, and Probability**
 5. **Algebra**



First Task: Judging Comparability at Content-Area Level

1. Review the SAT descriptors and NAEP objectives
2. Take out your judgment form
3. Read the first SAT descriptor



First Task *(continued)*

4. Then refer to the NAEP objectives to determine if the SAT descriptor is covered by NAEP
 - **Yes or No**
5. If you responded “yes,” then judge the strength of the NAEP-SAT comparability for that SAT descriptor:
 - **Weak, Moderate, or Strong**
6. If you responded “yes,” then identify the specific NAEP objective(s) that cover the SAT descriptor
 - **Write down the NAEP objective number(s)**



Let's Look at the Rating Form

SAT Mathematics Descriptor	NAEP Objective(s)	Round 1 Ratings				
		Overall Coverage	Level of Comparability*			
A Number and Operations						
1	Identify factors of whole numbers		No	1	2	3
2	Solve word problems using addition, subtraction, multiplication and division of whole numbers	Yes Yes	No	1	2	3
3	Recall basic mathematical facts/definitions about exponential notation, including scientific notation	Yes	No	1	2	3



After You Provide Your Content-Area Ratings

- We will collect your content-area judgments and summarize them
- We will then share the summary and ask you to discuss the results and to share your perspectives
 - For which descriptors does there seem to be more or less convergence of judgment?
 - What led you to see or not see comparability?



After You Provide Your Content-Area Ratings *(continued)*

- After discussion, consider if you want to revise one or more of your judgments
 - You are not required to change your judgments, but this is your opportunity to do so.



Evaluation Forms

1. You will now complete an initial evaluation form, on which you'll indicate if you're ready to proceed.
2. At the end of the day, you'll complete a final evaluation form regarding your experience with the process.

Are there any questions before we hand out the first evaluation form?



Let's Rate the First 3 Descriptors and Then Discuss

SAT Mathematics Descriptor	NAEP Objective(s)	Round 1 Ratings				
		Overall Coverage	Level of Comparability*			
<i>A Number and Operations</i>						
1	Identify factors of whole numbers	No	1	2	3	
2	Solve word problems using addition, subtraction, multiplication and division of whole numbers	Yes Yes	No	1	2	3
3	Recall basic mathematical facts/definitions about exponential notation, including scientific notation	Yes	No	1	2	3



Completion of Remaining Ratings

- Now complete the ratings for the remaining SAT descriptors.
- Turn your table tent sideways when you are done, and we will collect your rating form.



Discussion of Ratings

- I'll now show you a summary of the ratings.
- We will discuss those descriptors for which there was the most variance in ratings.
- Please take notes and revise your ratings if you would like (there is no requirement to do so).
- Enter the second-round ratings in the applicable column.
- You do not need to end a second-round rating if it has not changed from the first.



Second Task: Overall Ratings

- You will now be asked to respond to 3 overall questions:
 - Whether the two tests cover the same types of Mathematics skills
 - **1 (Strongly Agree) to 4 (Strongly Disagree)**
 - Whether the two tests cover the same range/breadth of Mathematics skills
 - **1 (Strongly Agree) to 4 (Strongly Disagree)**
 - Based on your evaluation of comparability, do you think that a follow-up study comparing items on each test is justified?
 - **Yes or No**



Summary of Overall Ratings

- I'll now summarize for you the overall ratings.



Final Evaluation Form

- We'll now hand out the final evaluation form.
- Then we'll have a debriefing.



Thank You!

- We really appreciate your taking the time to participate in this study on such notice.
- Thank you so much for your efforts today.

Appendix G
Content-Area Rating Form

Panelist #: _____

NAEP/SAT Comparability Study Mathematics

Panelist Rating Form—Content-Area Ratings

On this form you will provide two types of ratings.

- The first rating is whether the content described by the SAT mathematics descriptor is covered by objective(s) in the NAEP framework. This is a yes or no rating.
- The second rating should be made only for those SAT descriptors that you have rated as a “yes,” that they are covered by objective(s) in the NAEP framework. This is a rating of the extent to which the NAEP objective(s) cover the same range of content/topics as the SAT descriptors, and is on a 3-point scale. A rating of “1” indicates weak alignment, “2” indicates moderate alignment, and “3” indicates strong alignment.
- A column is also provided in which to indicate which NAEP objective(s) cover the same content as the SAT descriptor. For example, an indication of *1d* would indicate that this NAEP objective—*Number Sense: Represent, interpret or compare expressions for real numbers, including expressions utilizing exponents and logarithms*—covers the same content as the given SAT descriptor.

Panelist Rating Form—Content-Area Ratings Mathematics

SAT Mathematics Descriptor		NAEP Objective(s)	Round 1 Ratings						Round 2 Ratings					
			Overall Coverage		Level of Comparability*			Overall Coverage		Level of Comparability*				
<i>A Number and Operations</i>														
1	Identify factors of whole numbers		Yes	No	1	2	3	Yes	No	1	2	3		
2	Solve word problems using addition, subtraction, multiplication and division of whole numbers		Yes	No	1	2	3	Yes	No	1	2	3		
3	Recall basic mathematical facts/definitions about exponential notation, including scientific notation		Yes	No	1	2	3	Yes	No	1	2	3		
4	Identify a rule that describes a numerical pattern in a sequence		Yes	No	1	2	3	Yes	No	1	2	3		
5	Identify, use, and represent fractions and percents in arithmetic and algebraic settings		Yes	No	1	2	3	Yes	No	1	2	3		
6	Use properties of even and odd numbers, multiples, and factors		Yes	No	1	2	3	Yes	No	1	2	3		
7	Identify and use the names for place values in solving problems involving decimal representations (e.g., tenths and hundredths)		Yes	No	1	2	3	Yes	No	1	2	3		
8	Use properties of inequalities to compare and order numbers		Yes	No	1	2	3	Yes	No	1	2	3		
9	Solve problems using ideas from basic set theory and basic number theory		Yes	No	1	2	3	Yes	No	1	2	3		

*Rating Scale for Level of Comparability

1=Weak
2=Moderate
3=Strong

SAT Mathematics Descriptor		NAEP Objective(s)	Round 1 Ratings			Round 2 Ratings						
			Overall Coverage	Level of Comparability*		Overall Coverage	Level of Comparability*					
A <i>Number and Operations (continued)</i>												
10	Recognize and apply ratio, proportion, or percent in solving problems		Yes	No	1	2	3	Yes	No	1	2	3
11	Use properties of real number operations, ordering, and the zero-product property		Yes	No	1	2	3	Yes	No	1	2	3
12	Solve problems involving counting techniques		Yes	No	1	2	3	Yes	No	1	2	3
13	Determine values or properties of numbers in a sequence when given a description of the sequence		Yes	No	1	2	3	Yes	No	1	2	3
14	Create and use ratios, fractions, or percents in solving problems		Yes	No	1	2	3	Yes	No	1	2	3
15	Solve more-complex counting problems (e.g., permutations, combinations, and inclusion/exclusion)		Yes	No	1	2	3	Yes	No	1	2	3
16	Use π in algebraic and geometric contexts		Yes	No	1	2	3	Yes	No	1	2	3
17	Create and use ratios, fractions, or percents, including algebraic expressions, in solving problems		Yes	No	1	2	3	Yes	No	1	2	3

*Rating Scale for Level of Comparability

- 1=Weak
- 2=Moderate
- 3=Strong

SAT Mathematics Descriptor		NAEP Objective(s)	Round 1 Ratings			Round 2 Ratings						
			Overall Coverage	Level of Comparability*		Overall Coverage	Level of Comparability*					
<i>B Algebra and Functions</i>												
1	Use letters as placeholders for unknown values		Yes	No	1	2	3	Yes	No	1	2	3
2	Treat expressions such as $a + b$ as a single quantity in linear problem situations (e.g., solving $2(a + b) = 6$ to find the value of $a + b$)		Yes	No	1	2	3	Yes	No	1	2	3
3	Verify that a value is a solution to a linear or quadratic equation (e.g., substitute and simplify)		Yes	No	1	2	3	Yes	No	1	2	3
4	Use function notation in simple situations (e.g., evaluation)		Yes	No	1	2	3	Yes	No	1	2	3
5	Use variables in multistep abstract settings (e.g., apply the distributive property across several variables)		Yes	No	1	2	3	Yes	No	1	2	3
6	Solve problems involving positive-integer exponents		Yes	No	1	2	3	Yes	No	1	2	3
7	Solve word problems involving linear relationships		Yes	No	1	2	3	Yes	No	1	2	3
8	Substitute values in and simplify systems of equations in two variables		Yes	No	1	2	3	Yes	No	1	2	3
9	Solve two-step linear equations		Yes	No	1	2	3	Yes	No	1	2	3
10	Evaluate an operation in two variables represented by unfamiliar symbols		Yes	No	1	2	3	Yes	No	1	2	3
11	Formulate and solve problems involving proportions		Yes	No	1	2	3	Yes	No	1	2	3

*Rating Scale for Level of Comparability

1=Weak
2=Moderate
3=Strong

SAT Mathematics Descriptor		NAEP Objective(s)	Round 1 Ratings						Round 2 Ratings					
			Overall Coverage		Level of Comparability*			Overall Coverage		Level of Comparability*				
<i>B Algebra and Functions (continued)</i>														
12	Solve multistep problems involving linear and quadratic relationships		Yes	No	1	2	3		Yes	No	1	2	3	
13	Use and interpret graphs, including graphs of step functions		Yes	No	1	2	3		Yes	No	1	2	3	
14	Solve problems involving algebraic inequalities		Yes	No	1	2	3		Yes	No	1	2	3	
15	Solve problems involving exponential growth and decay		Yes	No	1	2	3		Yes	No	1	2	3	
16	Evaluate an operation in three variables represented by unfamiliar symbols		Yes	No	1	2	3		Yes	No	1	2	3	
17	Apply the concept of absolute value to algebraic expressions		Yes	No	1	2	3		Yes	No	1	2	3	
18	Identify and analyze the qualitative behavior of graphs of nonlinear functions		Yes	No	1	2	3		Yes	No	1	2	3	
19	Solve problems involving nonlinear functions and equations (e.g., quadratic, exponential, and rational)		Yes	No	1	2	3		Yes	No	1	2	3	
20	Solve problems involving fractional and negative exponents		Yes	No	1	2	3		Yes	No	1	2	3	
21	Identify solution sets in algebraic situations involving inequalities		Yes	No	1	2	3		Yes	No	1	2	3	
22	Solve problems involving composition of functions (e.g., use the output of one function as the input to be evaluated in a second function)		Yes	No	1	2	3		Yes	No	1	2	3	

*Rating Scale for Level of Comparability

- 1=Weak
- 2=Moderate
- 3=Strong

SAT Mathematics Descriptor		NAEP Objective(s)	Round 1 Ratings						Round 2 Ratings					
			Overall Coverage		Level of Comparability*			Overall Coverage		Level of Comparability*				
<i>B Algebra and Functions (continued)</i>														
23	Solve problems involving variables with operations represented by unfamiliar symbols		Yes	No	1	2	3		Yes	No	1	2	3	
24	Generalize an exponential pattern from a geometric sequence		Yes	No	1	2	3		Yes	No	1	2	3	
25	Solve for one variable or expression in terms of another		Yes	No	1	2	3		Yes	No	1	2	3	
26	Work with systems of equations involving three or more variables		Yes	No	1	2	3		Yes	No	1	2	3	
27	Solve problems involving complex fractions		Yes	No	1	2	3		Yes	No	1	2	3	
28	Solve problems involving functions defined with unfamiliar symbols in one or more variables		Yes	No	1	2	3		Yes	No	1	2	3	
29	Identify, apply, and represent transformations of functions, graphically and algebraically (e.g., vertical shift)		Yes	No	1	2	3		Yes	No	1	2	3	
30	Apply properties of non-integer exponents		Yes	No	1	2	3		Yes	No	1	2	3	
31	Solve multistep problems involving algebraic inequalities		Yes	No	1	2	3		Yes	No	1	2	3	
32	Solve word problems involving rate of change in nonlinear or piecewise-linear settings		Yes	No	1	2	3		Yes	No	1	2	3	
33	Identify and use the relationship between the slope of a line and algebraic rate of change		Yes	No	1	2	3		Yes	No	1	2	3	

*Rating Scale for Level of Comparability

1=Weak
2=Moderate
3=Strong

SAT Mathematics Descriptor		NAEP Objective(s)	Round 1 Ratings					Round 2 Ratings					
			Overall Coverage		Level of Comparability*			Overall Coverage		Level of Comparability*			
<i>B Algebra and Functions (continued)</i>													
34	Interpret and solve word problems using multistep proportional reasoning		Yes	No	1	2	3		Yes	No	1	2	3
35	Transform an equation or expression by raising it to a power		Yes	No	1	2	3		Yes	No	1	2	3

*Rating Scale for Level of Comparability

- 1=Weak
- 2=Moderate
- 3=Strong

SAT Mathematics Descriptor		NAEP Objective(s)	Round 1 Ratings			Round 2 Ratings						
			Overall Coverage	Level of Comparability*		Overall Coverage	Level of Comparability*					
C <i>Geometry and Measurement</i>												
1	Solve geometry problems involving basic shapes (e.g., triangles, circles, and segments)		Yes	No	1	2	3	Yes	No	1	2	3
2	Recall basic mathematical facts about triangles (e.g., properties of isosceles triangles and the 180° angle sum property)		Yes	No	1	2	3	Yes	No	1	2	3
3	Apply properties of triangles, including congruence		Yes	No	1	2	3	Yes	No	1	2	3
4	Apply angle relationships, including those in polygons and circles		Yes	No	1	2	3	Yes	No	1	2	3
5	Solve problems involving the length of line segments		Yes	No	1	2	3	Yes	No	1	2	3
6	Recognize and use the following: <ul style="list-style-type: none"> • Simple inscribed and circumscribed figures • The Pythagorean Theorem • Coordinate geometry (e.g., slope calculations) • Parallelism and perpendicularity • Two- and three-dimensional figures • Figures composed of two or more simple shapes 		Yes	No	1	2	3	Yes	No	1	2	3
7	Interpret and solve two-step problems involving geometric proportions		Yes	No	1	2	3	Yes	No	1	2	3
8	Recognize and use volume in solving multistep problems		Yes	No	1	2	3	Yes	No	1	2	3

*Rating Scale for Level of Comparability

- 1=Weak
- 2=Moderate
- 3=Strong

SAT Mathematics Descriptor		NAEP Objective(s)	Round 1 Ratings						Round 2 Ratings									
			Overall Coverage		Level of Comparability*			Overall Coverage		Level of Comparability*								
C													<i>Geometry and Measurement (continued)</i>					
9	Use the relationships between the slopes of parallel and perpendicular lines in the coordinate plane		Yes	No	1	2	3		Yes	No	1	2	3					
10	Determine the effect of changes in the linear dimensions of a figure on other measures of the figure, such as area		Yes	No	1	2	3		Yes	No	1	2	3					
11	Interpret and solve multistep problems involving geometric proportions		Yes	No	1	2	3		Yes	No	1	2	3					
12	Solve problems involving networks		Yes	No	1	2	3		Yes	No	1	2	3					

*Rating Scale for Level of Comparability

- 1=Weak
- 2=Moderate
- 3=Strong

SAT Mathematics Descriptor		NAEP Objective(s)	Round 1 Ratings			Round 2 Ratings							
			Overall Coverage		Level of Comparability*			Overall Coverage		Level of Comparability*			
<i>D Data, Statistics, and Probability</i>													
1	Read simple data displays (e.g., bar graphs, line graphs, pictograms, and tables)		Yes	No	1	2	3		Yes	No	1	2	3
2	Read and interpret bar graphs		Yes	No	1	2	3		Yes	No	1	2	3
3	Extract and use relevant information from tables, graphs, and diagrams		Yes	No	1	2	3		Yes	No	1	2	3
4	Interpret and solve problems involving data displays (e.g., circle graphs)		Yes	No	1	2	3		Yes	No	1	2	3
5	Interpret and solve multistep problems involving data displays		Yes	No	1	2	3		Yes	No	1	2	3
6	Interpret the effect of changes in data on measures of center		Yes	No	1	2	3		Yes	No	1	2	3
7	Solve problems involving probability		Yes	No	1	2	3		Yes	No	1	2	3
8	Solve conditional probability problems		Yes	No	1	2	3		Yes	No	1	2	3
9	Solve geometric probability problems		Yes	No	1	2	3		Yes	No	1	2	3

*Rating Scale for Level of Comparability

- 1=Weak
- 2=Moderate
- 3=Strong

SAT Mathematics Descriptor		NAEP Objective(s)	Round 1 Ratings			Round 2 Ratings		
			Overall Coverage	Level of Comparability*		Overall Coverage	Level of Comparability*	
<i>E Problem Solving</i>								
1	Set up and solve one-step problems involving rates		Yes	No	1	2	3	Yes No 1 2 3
2	Apply a simple procedure to solve an arithmetic problem		Yes	No	1	2	3	Yes No 1 2 3
3	Solve one-step proportional reasoning problems		Yes	No	1	2	3	Yes No 1 2 3
4	Read, extract, and use relevant information from written descriptions and geometric figures to solve a problem		Yes	No	1	2	3	Yes No 1 2 3
5	Solve some multistep routine problems		Yes	No	1	2	3	Yes No 1 2 3
6	Solve problems involving rates and unit conversions		Yes	No	1	2	3	Yes No 1 2 3
7	Use multistep strategies to solve a problem, such as the following: <ul style="list-style-type: none"> • Drawing auxiliary lines • Breaking a larger problem down to smaller components 		Yes	No	1	2	3	Yes No 1 2 3
8	Solve multistep nonroutine problems (e.g., by trial and error)		Yes	No	1	2	3	Yes No 1 2 3
9	Solve multistep geometry problems involving the following: <ul style="list-style-type: none"> • Angle measures and relationships • Triangles 		Yes	No	1	2	3	Yes No 1 2 3

*Rating Scale for Level of Comparability

- 1=Weak
- 2=Moderate
- 3=Strong

SAT Mathematics Descriptor		NAEP Objective(s)	Round 1 Ratings			Round 2 Ratings							
			Overall Coverage		Level of Comparability*			Overall Coverage		Level of Comparability*			
<i>E Problem Solving (continued)</i>													
10	Solve problems using multiple strategies, including the following: <ul style="list-style-type: none"> • Visualization • Estimation skills • Recognizing relevant information • Function notation 		Yes	No	1	2	3		Yes	No	1	2	3
11	Use insight in solving nonroutine geometric problems involving the following: <ul style="list-style-type: none"> • Triangles • Patterns • Perimeter • The Pythagorean Theorem • Properties of circles 		Yes	No	1	2	3		Yes	No	1	2	3
12	Solve the first stage of a problem, and then apply that solution to solve the next stage of the problem		Yes	No	1	2	3		Yes	No	1	2	3
13	Recognize complexity in problems that appear at first to be routine		Yes	No	1	2	3		Yes	No	1	2	3
14	Develop and apply an effective strategy and keep track of information in solving a nonroutine problem		Yes	No	1	2	3		Yes	No	1	2	3
15	Identify relevant and irrelevant information when choosing a solution strategy		Yes	No	1	2	3		Yes	No	1	2	3
16	Solve multistep problems involving properties of integers		Yes	No	1	2	3		Yes	No	1	2	3

*Rating Scale for Level of Comparability

- 1=Weak
- 2=Moderate
- 3=Strong

SAT Mathematics Descriptor		NAEP Objective(s)	Round 1 Ratings			Round 2 Ratings						
			Overall Coverage		Level of Comparability*			Overall Coverage		Level of Comparability*		
<i>F Representation</i>												
1	Read pictorial and tabular representations to identify an answer		Yes	No	1	2	3	Yes	No	1	2	3
2	Select an appropriate representation for a proportion		Yes	No	1	2	3	Yes	No	1	2	3
3	Translate verbal statements into algebraic expressions		Yes	No	1	2	3	Yes	No	1	2	3
4	Create and apply an appropriate representation for a rate		Yes	No	1	2	3	Yes	No	1	2	3
5	Visualize or create a geometric representation to solve a problem		Yes	No	1	2	3	Yes	No	1	2	3
6	Translate between verbal and symbolic representations of linear expressions		Yes	No	1	2	3	Yes	No	1	2	3
7	Translate between equivalent symbolic representations of linear expressions		Yes	No	1	2	3	Yes	No	1	2	3
8	Recognize and translate among information represented verbally, graphically, numerically, and symbolically		Yes	No	1	2	3	Yes	No	1	2	3
9	Visualize or sketch a figure based on a verbal description to solve a problem		Yes	No	1	2	3	Yes	No	1	2	3
10	Interpret functions and graphs as models in applied situations		Yes	No	1	2	3	Yes	No	1	2	3
11	Translate verbal descriptions into algebraic representations in solving complex problems		Yes	No	1	2	3	Yes	No	1	2	3

*Rating Scale for Level of Comparability

- 1=Weak
- 2=Moderate
- 3=Strong

SAT Mathematics Descriptor		NAEP Objective(s)	Round 1 Ratings			Round 2 Ratings							
			Overall Coverage		Level of Comparability*			Overall Coverage		Level of Comparability*			
<i>F Representation (continued)</i>													
12	Translate among equivalent representations of expressions involving exponents		Yes	No	1	2	3		Yes	No	1	2	3
13	Compare and contrast algebraic and geometric representations		Yes	No	1	2	3		Yes	No	1	2	3
14	Translate verbal descriptions into nonlinear algebraic representations in solving complex problems		Yes	No	1	2	3		Yes	No	1	2	3

*Rating Scale for Level of Comparability

- 1=Weak
- 2=Moderate
- 3=Strong

SAT Mathematics Descriptor		NAEP Objective(s)	Round 1 Ratings			Round 2 Ratings						
			Overall Coverage		Level of Comparability*			Overall Coverage		Level of Comparability*		
G Reasoning												
1	Apply reasoning in solving straightforward problems in familiar settings		Yes	No	1	2	3	Yes	No	1	2	3
2	Reason about, structure, and solve problems about rates and proportions		Yes	No	1	2	3	Yes	No	1	2	3
3	Consider and compare different cases in reasoning about a problem situation		Yes	No	1	2	3	Yes	No	1	2	3
4	Make and test conjectures involving basic logic and set theory		Yes	No	1	2	3	Yes	No	1	2	3
5	Use basic number theory to investigate conjectures (e.g., conjectures about odd/even, positive/negative, and consecutive integers)		Yes	No	1	2	3	Yes	No	1	2	3
6	Recognize and use counterexamples		Yes	No	1	2	3	Yes	No	1	2	3
7	Consider multiple cases		Yes	No	1	2	3	Yes	No	1	2	3
8	Investigate and coordinate multiple conjectures to draw a logical conclusion		Yes	No	1	2	3	Yes	No	1	2	3
9	Decide which cases to consider in order to reach a conclusion		Yes	No	1	2	3	Yes	No	1	2	3
10	Make and test conjectures about properties of operations represented by unfamiliar symbols		Yes	No	1	2	3	Yes	No	1	2	3

*Rating Scale for Level of Comparability

- 1=Weak
- 2=Moderate
- 3=Strong

SAT Mathematics Descriptor		NAEP Objective(s)	Round 1 Ratings			Round 2 Ratings						
			Overall Coverage		Level of Comparability*			Overall Coverage		Level of Comparability*		
<i>H Connections</i>												
1	Make connections between Data Analysis and Number and Operations (e.g., use numerical judgment in reading a simple data display)		Yes	No	1	2	3	Yes	No	1	2	3
2	Use variables in a geometric context (e.g., work with unknown angles identified by x and y)		Yes	No	1	2	3	Yes	No	1	2	3
3	Use variables in areas other than algebra		Yes	No	1	2	3	Yes	No	1	2	3
4	Use connections between areas of mathematics, such as the following: <ul style="list-style-type: none"> Algebra and geometry (e.g., connect geometric slope with an algebraic expression) Data and algebra (e.g., compute mean of algebraic expressions) Applying proportions in geometric situations 		Yes	No	1	2	3	Yes	No	1	2	3

*Rating Scale for Level of Comparability

- 1=Weak
- 2=Moderate
- 3=Strong

SAT Mathematics Descriptor		NAEP Objective(s)	Round 1 Ratings			Round 2 Ratings							
			Overall Coverage		Level of Comparability*			Overall Coverage		Level of Comparability*			
<i>H Connections (continued)</i>													
5	Use connections between areas of mathematics, such as the following: <ul style="list-style-type: none"> • Coordinate Geometry and Algebra • Number and Operations and Data, Statistics, and Probability • Number and Operations and Geometry • Number and Operations and Algebra • Data, Statistics, and Probability and Geometry and Measurement • Algebra and Functions and Data, Statistics, and Probability 		Yes	No	1	2	3		Yes	No	1	2	3
6	Solve nonroutine problems involving the application of concepts from the following: <ul style="list-style-type: none"> • Algebra and Functions and Number and Operations • Geometry and Measurement and Algebra and Functions • Data, Statistics, and Probability and Number and Operations 		Yes	No	1	2	3		Yes	No	1	2	3

*Rating Scale for Level of Comparability

- 1=Weak
- 2=Moderate
- 3=Strong

SAT Mathematics Descriptor		NAEP Objective(s)	Round 1 Ratings			Round 2 Ratings						
			Overall Coverage	Level of Comparability*		Overall Coverage	Level of Comparability*					
<i>I Communication</i>												
1	Use the following notation and terms: <ul style="list-style-type: none"> Factor (whole number) Radius 		Yes	No	1	2	3	Yes	No	1	2	3
2	Use the following notation and terms: <ul style="list-style-type: none"> Congruent angles 		Yes	No	1	2	3	Yes	No	1	2	3
3	Use the following notation and terms: <ul style="list-style-type: none"> Function notation Parallel 		Yes	No	1	2	3	Yes	No	1	2	3
4	Use the following notation and terms: <ul style="list-style-type: none"> Consecutive integers “NOT,” “CANNOT,” “must,” “which of the following” Arcs Angle bisector 		Yes	No	1	2	3	Yes	No	1	2	3
5	Use the following notation and terms: <ul style="list-style-type: none"> Median Random 		Yes	No	1	2	3	Yes	No	1	2	3
6	Use the following notations and terms: <ul style="list-style-type: none"> π Tangent (line to a circle; circle to a circle) “more than” Symmetry about the origin 		Yes	No	1	2	3	Yes	No	1	2	3

*Rating Scale for Level of Comparability

1=Weak
2=Moderate
3=Strong

Appendix H

Initial Evaluation/Ready-to-Proceed Form

Panelist #: _____

**NAEP/SAT Comparability Study
Mathematics**

Panelist Rating Form—Overall Ratings

Rating 1—Overall Level of Content Comparability

Please indicate below the degree to which you agree with the following statement:

Based on the descriptors and objectives, the content of the SAT and NAEP are comparable.

Strongly Agree
1

Agree
2

Disagree
3

Strongly Disagree
4

Rating 2—Overall Breadth of Content Coverage

Please indicate below the degree to which you agree with the following statement:

Based on the descriptors and objectives, the overall breadth of the SAT and NAEP are comparable.

Strongly Agree
1

Agree
2

Disagree
3

Strongly Disagree
4

Rating 3—Advisability of Future Study

Please indicate below whether you agree with the following statement:

There is sufficient overall overlap between the SAT and NAEP to justify conducting a more extensive alignment study at the item level.

Yes

No

Appendix I
Overall Rating Form

Panelist #: _____

SAT/NAEP Comparability Study Mathematics

Initial Evaluation/Ready-to-Proceed Form

The purpose of this evaluation form is to get your feedback about the adequacy of the explanations and preparation you have received in order to make your judgments of content-area comparability.

Please read each statement and place an "X" in the box to represent your response.

Statement	Rating			
	Strongly Agree	Agree	Disagree	Strongly Disagree
I understand the purpose of the study.				
The overview of the assessments was presented clearly.				
The steps that I am to follow to make my content-area ratings were presented clearly.				
I understand what I will be expected to do to complete my content-area ratings.				

I am ready to proceed and to make my first set of content-area comparability judgments.

Yes No

If no, what other information/explanations do you need before making your first set of judgments?

Date _____ Signature _____

Appendix J

Final Evaluation Form

Panelist #: _____

SATNAEP Comparability Study Mathematics

Final Evaluation Form

The purpose of this evaluation form is to get your feedback about the overall study.

Please read each statement and place an "X" in the box to represent your response.

Statement	Rating			
	Strongly Agree	Agree	Disagree	Strongly Disagree
The SAT and NAEP skill statements were sufficiently detailed to judge comparability.				
<i>Content-Area Comparability Ratings</i>				
The content-area rating form was easy to complete.				
The summary of our content-area ratings was presented clearly.				
The discussion of the summary of content-area ratings was informative.				
The process of completing the content-area ratings was easy to follow.				
<i>Overall Ratings</i>				
The overall rating form was easy to complete.				
The summary of our overall ratings was presented clearly.				
The discussion of the summary of overall ratings was informative.				
The process of completing the overall ratings was easy to follow.				

Panelist #: _____

What we should consider doing differently the next time we conduct this type of study?

Thank you!