National Assessment Governing Board

Content Alignment Studies of the 2009 National Assessment of Educational Progress for Grade 12 Reading and Mathematics with SAT and ACCUPLACER Assessments of these Subjects

Submitted: May 14, 2010

Interim Report: Comparative Analysis of the Test Blueprints and Specifications for 2009 NAEP Grade 12 Mathematics and SAT Mathematics

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This study was funded by the National Assessment Governing Board under Contract ED-NAG-09-C-0001.

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Introduction

WestEd has been contracted by the National Assessment Governing Board to study the extent to which the grade 12 National Assessment of Educational Progress (NAEP) is aligned in content and complexity to SAT and ACCUPLACER in reading and in mathematics. This project is part of the Governing Board's 12th grade preparedness initiative and will yield information on using the grade 12 NAEP to report on student preparedness for postsecondary activities. The overall study includes examination of the alignment between the grade 12 NAEP in reading and mathematics and the SAT and ACCUPLACER assessments in those subjects.

As described in the design document for this alignment study (Webb, 2009), the full study includes weeklong alignment coding sessions with replicate panels of content experts in reading and mathematics using the Web Alignment Tool. One component of the alignment study is an initial comparative analysis between the test blueprints and specifications for NAEP and each assessment, which occurs prior to the alignment coding sessions in order to inform advance expectations for alignment, as well as to raise potential alignment issues prior to item coding. The comparative analysis also provides an additional view of the alignment between the two assessments, which can be used in interpreting the item alignment analysis results, once completed. Interim reports comparing the reading framework for NAEP with reading specifications for SAT and ACCUPLACER and comparing the mathematics framework for NAEP with the ACCUPLACER mathematics specifications have been submitted to the Governing Board.

The results of this comparative analysis are intended to provide information about the similarities and differences in assessment design and administration between the 2009 NAEP grade 12 Mathematics Assessment and the SAT Mathematics Test (hereafter NAEP and SAT, respectively). WestEd's lead content facilitator for mathematics conducted this analysis. For NAEP, the *Mathematics Framework for the 2009 National Assessment of Educational Progress* (National Assessment Governing Board, 2008) and *Mathematics Assessment and Item Specifications for the NAEP 2009 Mathematics Assessment* (National Assessment Governing Board, 2007) were used in this analysis. The content and numbering of the NAEP objectives for use in item coding are taken from Exhibits 3–7; formatting has been added for use in the NAEP–SAT alignment study. The resulting format provides the NAEP objective numbering used in this report, and appears in the objective-level comparison table located in Appendix A. For SAT, the College Board provided test specifications containing categories, descriptions, and distributions of content; these specifications were used as the basis for the analysis and were supplemented by information available on the College Board's website.[1]

This document presents the results of the comparative analysis. To provide context for the analysis, the report begins with an overview of each assessment's Purpose and Use, Test Administration Procedures, and Resources Available to Students. Following is a detailed discussion of the findings of the content comparison in terms of Content Organization, Specificity of Content, and Student Performance. Subsequent sections discuss the Number,

[1] SAT specifications information included in this report is the property of the College Board.

Proportion, and Format of Items; Scoring Rubrics and Rules for Constructed-Response Items; and Grade Level Targeted by Items. The final section provides a Summary of Content Overlap and Implications for Item Alignment. Complete side-by-side comparison charts on which the analysis is based are provided as Appendix A. A list of the NAEP objectives not addressed in the SAT specifications is provided as Appendix B. The final decision rules applied in the study are provided as Appendix C.

Purpose and Use

NAEP and SAT are designed to measure the mathematics achievement of students at largely similar ages and grade levels. NAEP is administered to students in the 12th grade; SAT is administered to students who are entering or planning to enter college at the freshman level, typically students who are currently in 11th or 12th grade (College Board, 2010e).

Although both assessments measure the mathematics skills of students at similar ages and stages of academic progress, they serve different purposes for different audiences. NAEP, commonly referred to as "the Nation's Report Card," is administered to "representative samples" (National Center for Educational Statistics) of students across the country and provides results for "national, regional, state, district, and subgroup achievement in mathematics." NAEP does not provide results for individual students. The information yielded by NAEP is intended to help "the public, educators, and policymakers understand strengths and weaknesses in student performance and make informed decisions about education" (National Assessment Governing Board, 2009, p. v). SAT is a "standardized college entrance test" primarily designed to help "college admissions officers make fair and informed admissions decisions" about students' college readiness (College Board, 2010a). Therefore, SAT provides results measuring the reading skills of individual students.

Test Administration Procedures

NAEP is a paper-based assessment administered to "random samples of students designed to be representative of the nation, different regions of the country, participating states, and large urban districts" (National Assessment Governing Board, 2009, p. 2). The items on the NAEP mathematics test are distributed across multiple test booklets "using a matrix sampling design" (2007, p. 99), ensuring that not all students taking the assessment will receive the same booklets or items. Students are expected to spend approximately one hour taking the test.

The NAEP mathematics test framework indicates that accommodations for students with special needs that are routinely provided by schools for their own testing programs are allowed in NAEP. Customary accommodations include:

- One-on-one testing
- Small group testing
- Extended time
- Oral reading of directions
- Large-print booklets

- Bilingual English/Spanish booklets
- Use of an aide to transcribe responses (National Assessment Governing Board, 2008, p. 67)

According to information on the College Board's website, SAT is administered seven times a year in the United States at designated testing sites nationwide. All test sites are affiliated with educational institutions and proctored by professional staff. Students are given 70 minutes total to complete the 54 math items.

The College Board makes a variety of presentation, responding, timing/scheduling, and setting accommodations available to eligible students taking College Board assessments. For students with disabilities, these include the following:

- Recorded tests
- Braille and large-print versions of the tests
- Calculators
- Interpreters, qualified readers, or transcribers
- Screen display adjustments
- Other generally accepted and necessary means of making materials available to individuals with impairments (College Board, 2010c)

Examinees seeking accommodations must obtain approval from College Board's Services for Students with Disabilities prior to test administration (College Board, 2010d).

Resources Available to Students

The grade 12 NAEP mathematics assessment permits the limited use of certain approved tools. These tools include a spinner, folding card, ruler/protractor, and scientific calculator, all of which are provided to students for use during the test. Students are also permitted to bring and make restricted use of their own calculators, scientific or otherwise.[2] NAEP does not regulate the types of calculators students are allowed to use on the grade 12 test (National Assessment Governing Board, 2007, p. 100).

Calculator use is limited to "calculator blocks," sets of items that might prove difficult to solve without a calculator and for which calculator use has been approved. At each grade level, approximately two-thirds of the blocks measure students' mathematical knowledge and skills without access to a calculator; the other third of the blocks allow the use of a calculator. The intention of the Governing Board is that no items on the 2009 NAEP at grade 12 are designed to provide an advantage to students with a graphing calculator. Estimated time required for any item should be based on the assumption that students are not using graphing calculators (p. 101).

[2] The intent of restrictions on calculator use at grade 12 is (1) to help ensure that items in calculator blocks cannot be solved in ways that are inconsistent with the knowledge and skills the items are intended to measure and (2) to maintain the security of NAEP test materials. These restrictions address issues such as calculators with QWERTY keyboards, communication between students during testing, and the use of stored formulas, algorithms, and other procedures (National Assessment Governing Board, 2007, pp. 100-101).

For SAT, students may use two No. 2 pencils (no pens or mechanical pencils), acceptable calculators, and watches without audible alarms. Students may not use cell phones, pagers, personal digital assistants, iPods, MP3 players, or any other digital or electronic equipment; scratch paper, notes, books, dictionaries, compasses, protractors, rulers, or any other aids; highlighters or colored pencils, portable listening or recording devices, cameras or other photographic equipment, timers, or watches with audible alarms. Students are not permitted to use cell phones during the breaks.[3]

Content Organization of NAEP and SAT

The content specifications of NAEP and SAT are organized hierarchically, in two (SAT) or three (NAEP) levels of increasing specificity. For this study—for clarity of discussion, and for consistency with the nomenclature used in the Web alignment tool—the three levels of the NAEP specifications, from broadest to most specific, will be referred to as standards, goals, and objectives, respectively. The two levels of the SAT specifications, from broadest to most specific, will be referred to as goals and objectives, respectively.

The NAEP framework is organized into five standards: Number Properties and Operations; Measurement; Geometry; Data Analysis, Statistics, and Probability; and Algebra. These standards, however, are not discrete or "fragmented" sets. While the structure of these standards helps classify the mathematical content of the NAEP assessment, it is expected "that the objectives and test items built on them will, in many cases, cross content area boundaries" (National Assessment Governing Board, 2008, p. 7).

The five NAEP standards and 24 goals are listed below. Counts of objectives for each standard are listed, and the full text of the 130 objectives is included in Appendix A.

1. Number Properties and Operations (20 objectives)
    - 1.1 Number sense
    - 1.2 Estimation
    - 1.3 Number operations
    - 1.4 Ratios and proportional reasoning
    - 1.5 Properties of number and operations
    - 1.6 Mathematical reasoning using number
2. Measurement (18 objectives)
    - 2.1 Measuring physical attributes
    - 2.2 Systems of measurement
    - 2.3 Measurement in triangles
3. Geometry (30 objectives)
    - 3.1 Dimension and shape

[3] Adapted from *Tips for Your Students*, available at http://professionals.collegeboard.com/testing/sat-reasoning/test-day/tips.

3.2 Transformation of shapes and preservation of properties

3.3 Relationships between geometric figures

3.4 Position, direction, and coordinate geometry

3.5 Mathematical reasoning in geometry

4. Data Analysis, Statistics, and Probability (32 objectives)

4.1 Data representation

4.2 Characteristics of data sets

4.3 Experiments and samples

4.4 Probability

4.5 Mathematical reasoning with data

5. Algebra (30 objectives)

5.1 Patterns, relations, and functions

5.2 Algebraic representations

5.3 Variables, expressions, and operations

5.4 Equations and inequalities

5.5 Mathematical reasoning in algebra

The SAT specifications provided by the College Board for use in this analysis consist of four goals and 29 objectives. The SAT specifications are organized into these goals: Numbers and Operations; Algebra and Functions; Geometry and Measurement; and Data Analysis, Statistics, and Probability.

SAT goals and objectives are listed below.[4]

N. Numbers and Operations

N.1 Properties of integers (odd/even, primes, factors, GCF, LCM, properties of 0 and 1, positive/negative, place value), elementary number theory – modular arithmetic, Diophantine equations, divisibility, remainders

N.2 Rational numbers including equivalence of fractions and decimals, scientific notation, reciprocals, place value

N.3 Arithmetic word problems (those word problems that usually do not require an algebraic equation to solve, including word problems that involve percents and ratios)

N.4 Sequences and series (rule given explicitly) – finding specific term or sum of terms, exponential growth

N.5 Sets (union, intersection, elements)

N.6 Counting problems (permutations, combinations, multiplication principal)

[4] Source: Derived from data provided by the College Board.

N.7 Logic/logical reasoning

A. Algebra and Functions

A.1 Operations with real numbers

A.2 Algebraic representations, translation, and algebraic word problems (those that usually require an algebraic equation to solve)

A.3 Linear equations and inequalities

A.4 Systems of linear equations and inequalities

A.5 Rational equations and inequalities

A.6 Radical equations

A.7 Quadratics – factoring difference of two squares, perfect square trinomials, and $ax^2 + bx + c$ trinomials (that are factorable), solving factorable quadratics

A.8 Manipulation with integer and rational exponents and using rules for exponents

A.9 Direct and inverse variation

A.10 Basic concepts of algebraic functions

- Using new definitions as operations
- Function notation and evaluation
- Functions as models
- Qualitative behavior of functions
- Coordinate – linear equations
- Coordinate – quadratic equations
- Concepts of domain and range
- Transformations

G. Geometry and Measurement

G.1 Points and lines in the plane (locus, parallel, perpendicular), including use of geometric notation (length, segments, lines, rays, and congruence)

G.2 Angles in the plane – including complementary, supplementary, right, vertical, and parallel lines

G.3 Triangles (nonspecial) – area, perimeter, ratio and comparison of measures, triangle inequality, interior and exterior angles, similar triangles (similarity added for the PSAT/NMSQT)

G.4 Special triangles (30-60-90, isosceles, equilateral, etc.) – including area, perimeter, ratio and comparison of measures, angles, Pythagorean theorem, similar triangles, and problems in which an alternate method of solution involves trigonometry

G.5 Circles – area, circumference, sectors, properties of tangent lines, polygons with circles

G.6 Polygons (without circles)

G.7 Solid geometric figures – cubes, prisms, cylinders, cones, spheres, pyramids (volume, surface area, properties)

G.8 Geometric Perception

G.9 Coordinate Geometry, including midpoint formula, distance formula, slope, and properties of parallel and perpendicular lines

D. Data Analysis, Statistics, and Probability

D.1 Data Interpretation (tables, charts, line graphs, circle graphs, histograms, pictographs, scatterplots and matrices)

D.2 Statistics – measures of central tendency (mean, median, mode) - (median and mode added for PSAT/NMSQT)

D.3 Probability – elementary, geometric

As is evident in the lists of content above, at the highest level, the five NAEP standards and the four SAT goals are nearly identical. The major difference between them regards their treatment of the content categories of Measurement and Geometry: whereas SAT combines them into a single goal (G. Geometry and Measurement), the NAEP standards split them into two goals (2. Measurement and 3. Geometry), each with its own set of objectives.[5] Generally, though, at the highest level, there is correspondence: each SAT goal has at least one objective that corresponds with a NAEP standard, goal, or objective, and each NAEP standard has at least one objective that corresponds with an SAT goal or objective.

At the objective level, the overlap is somewhat different. The 29 SAT objectives tend to correspond more closely to the 24 NAEP goals than they do to the 130 NAEP objectives. Although additional information is provided for many of the SAT objectives in the form of lists and parenthetical statements, the 130 NAEP objectives go beyond the 29 SAT objectives to more specifically delineate student expectations, incorporating many more specific topics, skills, and content elements than the SAT objectives. All but two SAT objectives (N.5 and A.6) correspond with at least one—and often multiple—NAEP objectives. (SAT D.1, for example, corresponds with nine NAEP objectives within the Data Analysis, Statistics, and Probability standard, again evincing the difference in levels of specificity between frameworks/specifications.)

Clear correspondence between some NAEP and SAT objectives, however, can be difficult to determine because a given SAT objective may correspond in a superficial way with many NAEP objectives. For example, the SAT objective G.8 (Geometric Perception) corresponds in varying degrees with 10 NAEP objectives in the Measurement and Geometry standards, and for most of those connections, the likelihood is that G.8 is assessed on NAEP in association with other Geometry and Measurement concepts, rather than as a discrete skill.

[5] It should be noted, however, that according to the NAEP test blueprint, Geometry and Measurement are collapsed together to comprise 30% of the complete assessment (National Assessment Governing Board, 2008, p. 6).

Indeed, given the difference in overall size and scope between NAEP and SAT, there may be only a few SAT objectives available to correspond with a much larger set of NAEP objectives within a given mathematics strand. For example, NAEP 1 (Number Properties and Operations) contains six goals and 20 objectives, whereas only eight SAT objectives, across two goals, address numbers (N.1, N.2, N.3, N.4, N.5, N.7, A.1, and A.8). A similar degree of coverage can be seen regarding the other four NAEP standards.

As indicated earlier, two SAT objectives do not correspond with any NAEP objective, although they do cover content contained within a broader NAEP goal and are typically considered to be at the grade level represented by NAEP.

- SAT N.5 ("Sets (union, intersection, elements)") does not correspond with any NAEP objectives but does correspond with NAEP goal 1.6. The content of this objective is often found in a standard addressing mathematical reasoning skills.
- SAT A.6 ("Radical equations") does not correspond with any NAEP objectives but does correspond with NAEP goal 5.4. The content of this objective is often found in a standard addressing equations.

It is not unusual to find that components of several SAT objectives are necessary to fully cover one NAEP objective. (For example, it takes seven of the nine SAT objectives within the Geometry and Measurement standard to correspond fully with NAEP 3.1.c.) Still, some NAEP objectives do not correspond with any of the SAT objectives. Examples of this include the NAEP objectives that address numeric estimation (1.2.b, 1.2.c, 1.2.d); systems of measurement (2.2.a, 2.2.b, 2.2.d, 2.2.e, 2.2.f); vectors, conic sections, and polar coordinates (3.4.e, 3.4.f, 3.4.g); geometric reasoning (3.5.a, 3.5.b, 3.5.c, 3.5.d, 3.5.e); and surveys (4.3.a, 4.3.b, 4.3.c, 4.3.d, 4.3.e). Four NAEP goals have no objectives that correspond with any SAT goal or objective: 1.2 ("Estimation"); 2.2 ("Systems of measurement"); 3.5 ("Mathematical reasoning in geometry"); and 4.3 ("Experiments and samples").

The NAEP mathematics framework is a substantially larger compilation of skills and knowledge—24 goals and 130 objectives for NAEP, compared with 4 goals and 29 objectives for SAT—and is more detailed and complex than the SAT mathematics specifications. The NAEP mathematics framework also incorporates many more specific topics, skills, and content elements than do the SAT mathematics specifications.

Specificity of Content

As indicated previously, NAEP objectives tend to be more narrowly focused than SAT objectives. As an example, NAEP 1.6.a, "Give a mathematical argument to establish the validity of a simple numerical property or relationship," provides more specificity regarding the nature of the task than the corresponding SAT objective, N.7, which simply states "Logic/logical reasoning." In some cases, however, NAEP objectives may include content that goes beyond even a combination of SAT objectives. For example, NAEP 1.1.d ("Represent, interpret, or compare expressions for real numbers, including expressions using exponents and logarithms") addresses the representation and interpretation of numbers and includes comparative expressions, while the two associated SAT objectives, N.2 and A.8 ("Rational numbers including equivalence of fractions and decimals, scientific notation, reciprocals, place value" and "Manipulation with

integer and rational exponents and using rules for exponents," respectively), address only the recognition of rational numbers.

As a further indication of the level of specificity described in the NAEP framework, the Governing Board has specified the intended distribution of cognitive complexity across NAEP items. According to the Governing Board, 25% of NAEP items are to be at a low level of mathematical complexity, 50% are to be at a medium level of complexity, and 25% are to be at a high level of complexity (National Assessment Governing Board, 2008, p. 38). The wording of NAEP objectives reflects this intended level of complexity, each clearly specifying the nature of the tasks to be performed. As examples of implied levels of cognitive complexity, one NAEP objective may require students to demonstrate reasoning about the mathematics described by an objective (a relatively sophisticated level of cognitive complexity), while another may require students to simply demonstrate proficiency in the skills related to an objective (a relatively low level of cognitive complexity). The NAEP levels of cognitive complexity are different from the Webb-developed depth of knowledge levels to be used in this study (Webb, 2009). However, the specificity of cognitive complexity intended by the distribution described above—and, more importantly, found in the text of the NAEP objectives themselves—allows the NAEP framework to be coded for depth of knowledge.

On the other hand, the SAT mathematics specifications do not provide this level of specificity with regard to complexity. At the most detailed level of the SAT specifications, objectives state a component of the corresponding goal but do not indicate the actual task to be completed in relation to the component. For example, SAT G.6 ("Polygons (without circles)") and G.9 ("Coordinate Geometry, including midpoint formula, distance formula, slope, and properties of parallel and perpendicular lines") do not indicate whether a student is to simply use a formula to determine a value (a relatively low level of cognitive complexity), apply geometric properties to determine the position of a geometric figure (a somewhat higher level of cognitive complexity), or perform a series of complex steps to solve a real-world problem (a relatively high level of cognitive complexity).

Due to this lack of specificity in the SAT specifications, it was determined by the Governing Board and WestEd that there was insufficient information in the SAT mathematics specifications for the objectives to be analyzed for their depth of knowledge level as part of the NAEP–SAT mathematics alignment study. The SAT test items can be coded for depth of knowledge, and the range of depth of knowledge can be reported, but the depth of knowledge consistency analysis cannot be conducted, given that the intended depth of knowledge level of the objectives cannot be reliably coded.

In summary, the NAEP objectives tend to describe the content more specifically than the SAT objectives and reflect a greater range of cognitive demand for the students.

Student Performance

The NAEP framework and the SAT specifications describe similar content. Both include objectives that require students to understand the concepts of numbers, measures, geometric properties, data, and algebra, and to be able to apply them through computation, interpretation, analysis, symbolic manipulation, modeling, and problem solving. However, even in areas of overlapping content, it appears that the two assessments differ in their expectations of student

performance. While the differences in the degree of specificity and phrasing of objectives make similarities and differences in expectation of student performance harder to identify, NAEP goals appear to require a higher degree of cognitive complexity than is required by comparable SAT objectives.

The differences in student performance expectations are also evident in the distribution of item types in each assessment. The NAEP framework calls for multiple-choice as well as short and extended constructed-response items, and the language of some of the NAEP objectives reflects the range of student performance possible in constructed-response item formats. For example, an item written to NAEP 3.5.c, "Analyze or explain a geometric argument by contradiction," would likely take the form of a constructed-response item. SAT also features both multiple-choice and constructed-response items, though in notably different proportions than NAEP. A discussion of these differences immediately follows.

Number, Proportion, and Format of Items

The test designs for NAEP and SAT differ greatly. There are 164 total mathematics items in the NAEP pool. NAEP is administered through a system of sampling; no single student completes all 164 items. Rather, each student completes two fixed "item sets" consisting of items drawn from the larger NAEP pool (National Assessment Governing Board, 2007, p. 99). Twenty-five items in the 2009 NAEP mathematics item pool reference set leaders. Of these 25, 13 are constructed-response items and 12 are multiple-choice items. Between two and four items can refer to a single set leader. There are nine set leaders total.

SAT is a fixed-form, paper-based assessment consisting of 10 separately timed sections. Each SAT mathematics test contains 44 multiple-choice items and 10 student-produced responses for a total of 54 items. Two complete mathematics tests were provided for the purposes of this study.

As discussed previously, the NAEP framework and SAT specifications differ in the proportion of items described. NAEP includes multiple-choice, short constructed-response, and extended constructed-response items. For NAEP, item distribution is expressed as the percentage of time students are expected to spend on each item type. For grade 12 mathematics, this distribution is divided equally between multiple-choice and constructed-response items (p. 97). In terms of the number of items in the total NAEP item pool to be included in the alignment study, the distribution is as follows: 66% (108) multiple-choice; 27% (45) short constructed-response; and 7% (11) extended constructed-response. SAT also consists of multiple-choice and constructed ("student-produced") response items. Of the items provided for use in this study, 81% (88) are multiple-choice items; 19% (20) are student-produced response items.

The NAEP specifications do not restrict difficulty or complexity levels to particular item types. While the NAEP specifications describe a tendency toward increased cognitive demand on the part of constructed-response items, they also indicate that any item type may assess "mathematics of greater or less depth and sophistication" (p. 47). SAT does not classify items according to difficulty or complexity, but rather by "abstract" and "concrete." Of the 108 items provided for use in this study, 74% (65) of multiple-choice and 65% (13) of student-produced response items are classified as "abstract"; the remaining 26% (23) and 35% (7), respectively, are classified as "concrete."

Scoring Rubrics and Rules for Constructed-Response Items

Multiple-choice items in NAEP are scored as correct or incorrect. Constructed-response items in NAEP are scored according to the following rubrics:

- short constructed-response items are scored dichotomously: "correct or incorrect";
- short constructed-response items are scored on a three-point scale: "correct, partial, or incorrect";
- extended constructed-response items may have up to five scoring categories: "extensive, satisfactory, partial, minimal, or incorrect." (National Assessment Governing Board, 2008, pp. 80-84)

Every constructed-response item in NAEP has its own scoring rubric written specifically for that item.

Items on SAT are scored as correct or incorrect. One point is added for each multiple-choice question answered correctly. For multiple-choice questions answered incorrectly, 1/4 point is subtracted; no points are subtracted for incorrect student-response questions or omitted questions (College Board, 2010b).

Grade Level Targeted by Items

According to the Governing Board, NAEP test questions should match the NAEP assessment framework and specifications, and accurately represent "the content domain to which inferences will be made" (National Assessment Governing Board, 2007, p. 116). "In broad terms, the [NAEP] framework attempts to answer the question: What mathematics skills should be assessed in 2009 on NAEP at grades 4, 8, and 12?" (2008, p. 2) Consistent with this driving question, the grade 12 NAEP test specifications for mathematics have been revised in recent years to improve NAEP's ability to report on how well prepared grade 12 students are for postsecondary education and training.

SAT is a "standardized college entrance test" primarily designed to help "college admissions officers make fair and informed admissions decisions" about students' college readiness (College Board, 2010a). SAT is administered to students who are entering or planning to enter college at the freshman level.

Summary of Content Overlap and Implications for Item Alignment

This section presents a summary of the overlap in content between the NAEP mathematics framework and the SAT mathematics specifications that was identified by the comparative analysis, indicating areas where the assessments appear to be have greater or lesser degrees of potential alignment. The comparative analysis has raised a number of considerations regarding issues that may arise in the alignment study. These issues are described here, along with proposed decision rules to address them, if necessary.

Summary of Content Overlap

Table 1 summarizes the overlap between SAT and NAEP objectives. Overlap, as shown in Table 1, means that one or more NAEP objective has been found to correspond to a given SAT objective. SAT is represented in the table at the objective level by goal, and NAEP is represented at the goal level by standard. Goals and standards represented in Table 1 are provided for summary purposes only. The complete comparative analysis table for the alignment of SAT to NAEP at the objective level, on which this summary is based, is included as Appendix A.

Table 1. Summary of Overlap of SAT and NAEP Objectives

SAT **(Objective level, organized by goal)**	**NAEP** **(Goal level, organized by standard)**
N. Numbers and Operations N.1 Properties of integers (odd/even, primes, factors, GCF, LCM, properties of 0 and 1, positive/negative, place value), elementary number theory – modular arithmetic, Diophantine equations, divisibility, remainders	**1. Number properties and operations** 1.5 Properties of number and operations **5. Algebra** 5.4 Equations and inequalities
N. Numbers and Operations N.2 Rational numbers including equivalence of fractions and decimals, scientific notation, reciprocals, place value	**1. Number properties and operations** 1.1 Number sense 1.5 Properties of number and operations
N. Numbers and Operations N.3 Arithmetic word problems (those word problems that usually do not require an algebraic equation to solve, including word problems that involve percents and ratios)	**1. Number properties and operations** 1.3 Number operations 1.4 Ratios and proportional reasoning
N. Numbers and Operations N.4 Sequences and series (rule given explicitly) – finding specific term or sum of terms, exponential growth	**5. Algebra** 5.1 Patterns, relations, and functions 5.3 Variables, expressions, and operations
N. Numbers and Operations N.5 Sets (union, intersection, elements)	**1. Number properties and operations** 1.6 Mathematical reasoning using number
N. Numbers and Operations N.6 Counting problems (permutations, combinations, multiplication principal)	**4. Data analysis, statistics, and probability** 4.4 Probability
N. Numbers and Operations N.7 Logic/logical reasoning	**1. Number properties and operations** 1.6 Mathematical reasoning using number **5. Algebra** 5.5 Mathematical reasoning in algebra
A. Algebra and Functions A.1 Operations with real numbers	**1. Number properties and operations** 1.3 Number operations
A. Algebra and Functions A.2 Algebraic representations, translation, and algebraic word problems (those that usually require an algebraic equation to solve)	**5. Algebra** 5.1 Patterns, relations, and functions 5.2 Algebraic representations 5.3 Variables, expressions, and operations 5.4 Equations and inequalities

SAT (Objective level, organized by goal)	NAEP (Goal level, organized by standard)
A. Algebra and Functions A.3 Linear equations and inequalities	**5. Algebra** 5.1 Patterns, relations, and functions 5.2 Algebraic representations 5.3 Variables, expressions, and operations 5.4 Equations and inequalities
A. Algebra and Functions A.4 Systems of linear equations and inequalities	**5. Algebra** 5.2 Algebraic representations 5.3 Variables, expressions, and operations 5.4 Equations and inequalities
A. Algebra and Functions A.5 Rational equations and inequalities	**5. Algebra** 5.1 Patterns, relations, and functions 5.2 Algebraic representations 5.3 Variables, expressions, and operations 5.4 Equations and inequalities
A. Algebra and Functions A.6 Radical equations	**5. Algebra** 5.4 Equations and inequalities
A. Algebra and Functions A.7 Quadratics – factoring difference of two squares, perfect square trinomials, and $ax^2 + bx + c$ trinomials (that are factorable), solving factorable quadratics	**5. Algebra** 5.2 Algebraic representations 5.4 Equations and inequalities
A. Algebra and Functions A.8 Manipulation with integer and rational exponents and using rules for exponents	**1. Number properties and operations** 1.1 Number sense 1.3 Number operations **5. Algebra** 5.3 Variables, expressions, and operations
A. Algebra and Functions A.9 Direct and inverse variation	**1. Number properties and operations** 1.3 Number operations **5. Algebra** 5.2 Algebraic representations 5.3 Variables, expressions, and operations
A. Algebra and Functions A.10 Basic concepts of algebraic functions: Using new definitions as operations; Function notation and evaluation; Functions as models; Qualitative behavior of functions; Coordinate – linear equations; Coordinate – quadratic equations; Concepts of domain and range; Transformations	**5. Algebra** 5.1 Patterns, relations, and functions 5.2 Algebraic representations 5.3 Variables, expressions, and operations
G. Geometry and Measurement G.1 Points and lines in the plane (locus, parallel, perpendicular), including use of geometric notation (length, segments, lines, rays, and congruence)	**3. Geometry** 3.1 Dimension and shape 3.3 Relationships between geometric figures 3.4 Position, direction, and coordinate geometry
G. Geometry and Measurement G.2 Angles in the plane – including complementary, supplementary, right, vertical, and parallel lines	**2. Measurement** 2.1 Measuring physical attributes **3. Geometry** 3.1 Dimension and shape 3.3 Relationships between geometric figures

SAT (Objective level, organized by goal)	NAEP (Goal level, organized by standard)
G. Geometry and Measurement G.3 Triangles (nonspecial) – area, perimeter, ratio and comparison of measures, triangle inequality, interior and exterior angles, similar triangles (similarity added for the PSAT/NMSQT)	**2. Measurement** 2.1 Measuring physical attributes 2.3 Measurement in triangles **3. Geometry** 3.1 Dimension and shape 3.3 Relationships between geometric figures
G. Geometry and Measurement G.4 Special triangles (30-60-90, isosceles, equilateral, etc.) – including area, perimeter, ratio and comparison of measures, angles, Pythagorean theorem, similar triangles, and problems in which an alternate method of solution involves trigonometry	**2. Measurement** 2.1 Measuring physical attributes 2.3 Measurement in triangles **3. Geometry** 3.1 Dimension and shape 3.3 Relationships between geometric figures
G. Geometry and Measurement G.5 Circles – area, circumference, sectors, properties of tangent lines, polygons with circles	**2. Measurement** 2.1 Measuring physical attributes **3. Geometry** 3.1 Dimension and shape 3.2 Transformation of shapes and preservation of properties 3.3 Relationships between geometric figures
G. Geometry and Measurement G.6 Polygons (without circles)	**3. Geometry** 3.1 Dimension and shape 3.2 Transformation of shapes and preservation of properties 3.3 Relationships between geometric figures 3.4 Position, direction, and coordinate geometry
G. Geometry and Measurement G.7 Solid geometric figures – cubes, prisms, cylinders, cones, spheres, pyramids (volume, surface area, properties)	**2. Measurement** 2.1 Measuring physical attributes **3. Geometry** 3.1 Dimension and shape 3.2 Transformation of shapes and preservation of properties 3.3 Relationships between geometric figures
G. Geometry and Measurement G.8 Geometric Perception	**2. Measurement** 2.1 Measuring physical attributes **3. Geometry** 3.1 Dimension and shape 3.2 Transformation of shapes and preservation of properties 3.4 Position, direction, and coordinate geometry
G. Geometry and Measurement G.9 Coordinate Geometry, including midpoint formula, distance formula, slope, and properties of parallel and perpendicular lines	**3. Geometry** 3.4 Position, direction, and coordinate geometry
D. Data Analysis, Statistics, and Probability D.1 Data Interpretation (tables, charts, line graphs, circle graphs, histograms, pictographs, scatterplots and matrices)	**4. Data analysis, statistics, and probability** 4.1 Data representation 4.2 Characteristics of data sets 4.5 Mathematical reasoning with data

SAT (Objective level, organized by goal)	NAEP (Goal level, organized by standard)
D. Data Analysis, Statistics, and Probability D.2 Statistics – measures of central tendency (mean, median, mode) - (median and mode added for PSAT/NMSQT)	**4. Data analysis, statistics, and probability** 4.2 Characteristics of data sets 4.5 Mathematical reasoning with data
D. Data Analysis, Statistics, and Probability D.3 Probability – elementary, geometric	**4. Data analysis, statistics, and probability** 4.4 Probability
NAEP goals not identified with any SAT goal/objective	**1. Number properties and operations** 1.2 Estimation **2. Measurement** 2.2 Systems of measurement **3. Geometry** 3.5 Mathematical reasoning in geometry **4. Data analysis, statistics, and probability** 4.3 Experiments and samples

As shown in Table 1, all of the content represented by the four SAT goals is represented by one or more NAEP goals. In some cases, the wording of an SAT objective is very similar to the wording in the related NAEP goal. In other cases, the content overlap is inferred.

Although every SAT objective has overlapping content with a NAEP standard, not every NAEP goal is represented in the SAT specifications. Of the 24 NAEP goals, four are not represented by an SAT objective. A more comprehensive comparison of the SAT objectives and the NAEP objectives is found in Appendix A. A list of NAEP objectives with no corresponding SAT objectives is located in Appendix B.

Implications for the Alignment Study and Related Potential Decision Rules

This section describes implications of the findings of this comparative analysis. Where potential decision rules have emerged, these are included below the related implication. The final decision rules used in the alignment study are included in Appendix C.

Considering items that contain operations involving only numbers and those involving variables
Objectives for the NAEP framework and SAT specifications consider representations and operations on both numbers and variables. Items containing numbers should be primarily aligned to those objectives addressing numbers, and items containing variables should be primarily aligned to those objectives addressing algebra. As an example, SAT A.8 ("Manipulation with integer and rational exponents and using rules for exponents") is included within SAT A (Algebra and Functions). One would expect to see variables in items corresponding with this objective. Therefore, based on this decision rule, one would not determine SAT A.8 to correspond with NAEP 1.1.d ("Represent, interpret, or compare expressions for real numbers, including expressions using exponents and logarithms"), as this NAEP objective would likely address numerical expressions and not variables.

Proposed Decision Rule: The NAEP objectives within the Algebra standard will be interpreted as aligning primarily to items containing one or more variables and not to items containing only numerical expressions. NAEP objectives within the Number Operations and Properties standard

will be interpreted as aligning primarily to numerical items; however, consideration is also to be given to items containing one or more variables.

Considering objectives that require the student to solve problems

The various NAEP objectives make reference to problem solving using wording that varies, but has similar connotations. Each of those objectives is interpreted to assess mathematics in situations involving either real-world problem solving or problem solving in a mathematical context. For example, it is determined that each of the following NAEP objectives implies problem solving tasks, even though the wording of each is somewhat different (italics added for emphasis):

- NAEP 1.1.g ("Represent, interpret, or compare expressions or *problem situations* involving absolute values")
- NAEP 1.2.c ("Verify solutions or determine the reasonableness of results in a *variety of situations*")
- NAEP 1.3.f ("*Solve application problems* involving numbers, including rational and common irrationals")
- NAEP 1.4.c ("Use proportions to *solve problems* (including rates of change)")
- NAEP 1.4.d ("*Solve multistep problems* involving percentages, including compound percentages")

Within the SAT specifications, it is determined that each of the following SAT objectives implies problem solving tasks (italics added for emphasis):

- SAT N.3 ("Arithmetic word problems (those *word problems* that usually do not require an algebraic equation to solve, including word problems that involve percents and ratios)")
- SAT A.2 ("Algebraic representations, translation, and algebraic *word problems* (those that usually require an algebraic equation to solve)")
- SAT G.4 ("Special triangles (30-60-90, isosceles, equilateral, etc.) – including area, perimeter, ratio and comparison of measures, angles, Pythagorean theorem, similar triangles, and *problems* in which an alternate method of solution involves trigonometry")

Proposed Decision Rule: The primary intent of objectives containing wording that refers to problem solving (e.g., solving problems, in contextual situations) is to assess mathematics in situations involving either real-world problem solving or problem solving in a mathematical context.

Considering objectives that have multiple parts

Some NAEP objectives contain multiple parts separated by the word "and." The intent of the objective is not necessarily interpreted to require that all parts assessed by each item align to that objective. For example, in NAEP 1.1.d ("Represent, interpret, or compare expressions for real numbers, including expressions using exponents and logarithms"), the "or" indicates that one would not be expected to do all three at once in the same item. The "and," however, could be interpreted as replaceable by an "or" (one would not use both exponents and logarithms). As another example, within NAEP 1.2.b ("Identify situations where estimation is appropriate,

determine the needed degree of accuracy, and analyze* the effect of the estimation method on the accuracy of results"), the "and" preceding "analyze the effect" makes the interpretation of this objective somewhat ambiguous. As written, this objective implies that an item could require a student to identify, determine, and analyze; however, this intent of this objective is interpreted as inclusive of items that focus on any one—or combination—of the three tasks.

Proposed Decision Rule: If an item addresses only one part of the objective, panelists are asked to look for an alternative primary code. If an alternative code is not available, panelists are to note in the WAT that the item does not assess the entire objective.

Considering objectives that assess equations

Some NAEP objectives specifically address algebraic manipulations involving expressions, and others specifically address equations and inequalities. Since every equation and inequality contains two expressions, it is understood that those algebraic manipulations include expressions as well. As an example, NAEP 5.3.c ("Perform basic operations, using appropriate tools, on algebraic expressions including polynomial and rational expressions") could be interpreted to include basic operations performed on equations as well as expressions; therefore, an SAT item that involves equations but not expressions would still be determined to correspond with this NAEP objective.

Proposed Decision Rule: An objective that addresses expressions may be aligned with items containing equations if symbolic manipulation across the equal sign is not required to answer the question. An objective that addresses equation or inequalities may or may not also address expressions.

Considering SAT objectives that include parenthetical statements

When considering the wording of an SAT objective that includes a parenthetical statement, panelists will consider the wording to be a clarification, rather than an exclusive list. For example, SAT N.1 ("Properties of integers (odd/even, primes, factors, GCF, LCM, properties of 0 and 1, positive/negative, place value), elementary number theory – modular arithmetic, Diophantine equations, divisibility, remainders") provides an "e.g." list of properties to consider. That list is not considered to necessarily exclude a NAEP item written to NAEP 1.5.f ("Recognize properties of the number system (whole numbers, integers, rational numbers, real numbers, and complex numbers) and how they are related to each other, and identify examples of each type of number") that is not specifically included in the parenthetical.

Proposed Decision Rule: If an SAT objective includes a parenthetical statement, panelists will consider the wording to be a clarification, rather than an exclusive list.

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Appendix A: Objective-Level NAEP-to-SAT Content Comparison

NAEP Mathematics	SAT Mathematics	Similarities and Differences
1 Number properties and operations	N. Numbers and Operations	
1.1 Number sense		
1.1.d Represent, interpret, or compare expressions for real numbers, including expressions using exponents and logarithms.	N.2 Rational numbers including equivalence of fractions and decimals, scientific notation, reciprocals, place value; A.8 Manipulation with integer and rational exponents and using rules for exponents	The NAEP objective addresses real numbers in a broad sense. The SAT objective only addresses a subset of real numbers at a basic level of understanding.
1.1.f Represent or interpret expressions involving very large or very small numbers in scientific notation.	N.2 Rational numbers including equivalence of fractions and decimals, scientific notation, reciprocals, place value	The NAEP objective only addresses the decimal number system concepts that involve scientific notation. The SAT objective addresses scientific notation equivalence, among other related concepts.
1.1.g Represent, interpret, or compare expressions or problem situations involving absolute values.		No SAT objectives align to NAEP 1.1.g.
1.1.i Order or compare real numbers, including very large and very small real numbers.	N.2 Rational numbers including equivalence of fractions and decimals, scientific notation, reciprocals, place value	The SAT objective infers ordering and comparing as it relates to equivalence.
1.2 Estimation		
1.2.b Identify situations where estimation is appropriate, determine the needed degree of accuracy, and analyze* the effect of the estimation method on the accuracy of results.		No SAT objectives align to NAEP 1.2.b.
1.2.c Verify solutions or determine the reasonableness of results in a variety of situations.		No SAT objectives align to NAEP 1.2.c.
1.2.d Estimate square or cube roots of numbers less than 1,000 between two whole numbers.		No SAT objectives align to NAEP 1.2.d.
1.3 Number operations		
1.3.a Find integral or simple fractional powers of real numbers.	A.8 Manipulation with integer and rational exponents and using rules for exponents	The NAEP objective and the SAT objective are equivalent.
1.3.b Perform arithmetic operations with real numbers, including common irrational numbers.	A.1 Operations with real numbers	The NAEP objective and the SAT objective are equivalent.
1.3.c Perform arithmetic operations with expressions involving absolute value.		No SAT objectives align to NAEP 1.3.c.

NAEP Mathematics	SAT Mathematics	Similarities and Differences
1.3.d Describe the effect of multiplying and dividing by numbers including the effect of multiplying or dividing a real number by: Zero, or A number less than zero, or A number between zero and one, or One, or A number greater than one.	A.1 Operations with real numbers	The NAEP objective specifically addresses the effects of real number operations. The SAT objective only addresses operations with real numbers in a broad sense.
1.3.f Solve application problems involving numbers, including rational and common irrationals.	N.3 Arithmetic word problems (those word problems that usually do not require an algebraic equation to solve, including word problems that involve percents and ratios); A.9 Direct and inverse variation	The NAEP objective addresses applications of problems with numbers in a broad sense. The SAT objectives involve solving real-world problems, but likely do not address the full range of application problems addressed by NAEP, and possibly not those involving irrational numbers.
1.4 Ratios and proportional reasoning		
1.4.c Use proportions to solve problems (including rates of change).	N.3 Arithmetic word problems (those word problems that usually do not require an algebraic equation to solve, including word problems that involve percents and ratios)	The NAEP objective and the SAT objective are equivalent.
1.4.d Solve multistep problems involving percentages, including compound percentages.	N.3 Arithmetic word problems (those word problems that usually do not require an algebraic equation to solve, including word problems that involve percents and ratios)	The NAEP objective and the SAT objective are equivalent.
1.5 Properties of number and operations		
1.5.c Solve problems using factors, multiples, or prime factorization.	N.1 Properties of integers (odd/even, primes, factors, GCF, LCM, properties of 0 and 1, positive/negative, place value), elementary number theory – modular arithmetic, Diophantine equations, divisibility, remainders	The NAEP objective addresses solving problems involving the properties of numbers. The SAT objective may include applications, but likely addresses number theory in a more general sense.
1.5.d Use divisibility or remainders in problem settings.	N.1 Properties of integers (odd/even, primes, factors, GCF, LCM, properties of 0 and 1, positive/negative, place value), elementary number theory – modular arithmetic, Diophantine equations, divisibility, remainders	The NAEP objective addresses solving problems involving divisibility. The SAT objective may include applications, but likely addresses number theory more than problem solving.
1.5.e Apply basic properties of operations, including conventions about the order of operations.		No SAT objectives align to NAEP 1.5.e.

NAEP Mathematics	SAT Mathematics	Similarities and Differences
1.5.f Recognize properties of the number system (whole numbers, integers, rational numbers, real numbers, and complex numbers) and how they are related to each other, and identify examples of each type of number.	N.1 Properties of integers (odd/even, primes, factors, GCF, LCM, properties of 0 and 1, positive/negative, place value), elementary number theory – modular arithmetic, Diophantine equations, divisibility, remainders; N.2 Rational numbers including equivalence of fractions and decimals, scientific notation, reciprocals, place value	The NAEP objective addresses properties of number systems in a broad sense. The SAT objectives are limited to the properties of integers and rational number equivalence.
1.6 Mathematical reasoning using number	N.5 Sets (union, intersection, elements)	The SAT objective is not specifically addressed by any NAEP objectives, but is related to the NAEP goal.
1.6.a Give a mathematical argument to establish the validity of a simple numerical property or relationship.	N.7 Logic/logical reasoning	The NAEP objective addresses reasoning as it specifically relates to the validity of a number property. The SAT objective more broadly addresses logical reasoning as it relates to numbers.
1.6.b * Analyze or interpret a proof by mathematical induction of a simple numerical relationship.	N.7 Logic/logical reasoning	The NAEP objective addresses reasoning as it specifically relates to the validity of a number property. The SAT objective more broadly addresses logical reasoning as it relates to numbers.
2 Measurement	G. Geometry and Measurement	The NAEP standard is addressed by the SAT goal that also addresses geometry.
2.1 Measuring physical attributes		
2.1.b Determine the effect of proportions and scaling on length, area, and volume.	G.3 Triangles (nonspecial) – area, perimeter, ratio and comparison of measures, triangle inequality, interior and exterior angles, similar triangles (similarity added for the PSAT/NMSQT); G.4 Special triangles (30-60-90, isosceles, equilateral, etc.) – including area, perimeter, ratio and comparison of measures, angles, Pythagorean theorem, similar triangles, and problems in which an alternate method of solution involves trigonometry; G.7 Solid geometric figures – cubes, prisms, cylinders, cones, spheres, pyramids (volume, surface area, properties)	The NAEP objective specifically addresses the effects of proportions and scaling. The SAT objectives more broadly address geometric concepts involving triangles and 3-D shapes, which likely include the more specific NAEP concept.
2.1.c Estimate or compare perimeters or areas of two-dimensional geometric figures.	G.8 Geometric Perception	The NAEP objective addresses estimation, whereas the SAT objective only addresses one aspect of estimation involving geometric figures: visualization.

NAEP Mathematics	SAT Mathematics	Similarities and Differences
2.1.d Solve problems of angle measure, including those involving triangles or other polygons or parallel lines cut by a transversal.	G.2 Angles in the plane – including complementary, supplementary, right, vertical, and parallel lines	The NAEP objective addresses problems involving angles within shapes as well as angles made by lines, whereas the SAT objective only involves angles formed by lines.
2.1.f Solve problems involving perimeter or area of plane figures such as polygons, circles, or composite figures.	G.3 Triangles (nonspecial) – area, perimeter, ratio and comparison of measures, triangle inequality, interior and exterior angles, similar triangles (similarity added for the PSAT/NMSQT); G.4 Special triangles (30-60-90, isosceles, equilateral, etc.) – including area, perimeter, ratio and comparison of measures, angles, Pythagorean theorem, similar triangles, and problems in which an alternate method of solution involves trigonometry; G.5 Circles – area, circumference, sectors, properties of tangent lines, polygons with circles	The NAEP objective addresses problem solving involving the area and perimeter of plane shapes. The SAT objectives address area and perimeter of only triangles and circles.
2.1.h Solve problems by determining, estimating, or comparing volumes or surface areas of three-dimensional figures.	G.7 Solid geometric figures – cubes, prisms, cylinders, cones, spheres, pyramids (volume, surface area, properties); G.8 Geometric Perception	The NAEP objective and the SAT objectives are equivalent.
2.1.i Solve problems involving rates such as speed, density, population density, or flow rates.		No SAT objectives align to NAEP 2.1.i.
2.2 Systems of measurement		
2.2.a Recognize that geometric measurements (length, area, perimeter, and volume) depend on the choice of a unit, and apply such units in expressions, equations, and problem solutions.		No SAT objectives align to NAEP 2.2.a.
2.2.b Solve problems involving conversions within or between measurement systems, given the relationship between the units.		No SAT objectives align to NAEP 2.2.b.
2.2.d Understand that numerical values associated with measurements of physical quantities are approximate, are subject to variation, and must be assigned units of measurement.		No SAT objectives align to NAEP 2.2.d.

NAEP Mathematics	SAT Mathematics	Similarities and Differences
2.2.e Determine appropriate accuracy of measurement in problem situations (e.g., the accuracy of measurement of the dimensions to obtain a specified accuracy of area) and find the measure to that degree of accuracy.		No SAT objectives align to NAEP 2.2.e.
2.2.f Construct or solve problems involving scale drawings.		No SAT objectives align to NAEP 2.2.f.
2.3 Measurement in triangles		
2.3.a Solve problems involving indirect measurement.	G.3 Triangles (nonspecial) – area, perimeter, ratio and comparison of measures, triangle inequality, interior and exterior angles, similar triangles (similarity added for the PSAT/NMSQT); G.4 Special triangles (30-60-90, isosceles, equilateral, etc.) – including area, perimeter, ratio and comparison of measures, angles, Pythagorean theorem, similar triangles, and problems in which an alternate method of solution involves trigonometry	The NAEP objective and the SAT objectives are equivalent.
2.3.b Solve problems using the fact that trigonometric ratios (sine, cosine, and tangent) stay constant in similar triangles.	G.4 Special triangles (30-60-90, isosceles, equilateral, etc.) – including area, perimeter, ratio and comparison of measures, angles, Pythagorean theorem, similar triangles, and problems in which an alternate method of solution involves trigonometry	The NAEP objective specifically addresses solving problems involving similar right triangles using trigonometry. The SAT objective more broadly addresses the geometric concepts involving right triangles.
2.3.c Use the definitions of sine, cosine, and tangent as ratios of sides in a right triangle to solve problems about length of sides and measure of angles.	G.4 Special triangles (30-60-90, isosceles, equilateral, etc.) – including area, perimeter, ratio and comparison of measures, angles, Pythagorean theorem, similar triangles, and problems in which an alternate method of solution involves trigonometry	See NAEP 2.3.b.
2.3.d Interpret and use the identity $\sin^2 \theta + \cos^2 \theta = 1$ for angles θ between 0° and 90°; recognize this identity as a special representation of the Pythagorean theorem.		No SAT objectives align to NAEP 2.3.d.
2.3.e * Determine the radian measure of an angle and explain how radian measurement is related to a circle of radius 1.		No SAT objectives align to NAEP 2.3.e.

NAEP Mathematics	SAT Mathematics	Similarities and Differences
2.3.f * Use trigonometric formulas such as addition and double angle formulas.		No SAT objectives align to NAEP 2.3.f.
2.3.g * Use the law of cosines and the law of sines to find unknown sides and angles of a triangle.	G.3 Triangles (nonspecial) – area, perimeter, ratio and comparison of measures, triangle inequality, interior and exterior angles, similar triangles (similarity added for the PSAT/NMSQT)	The NAEP objective specifically addresses using the law of cosines and the law of sines. The SAT objective more broadly addresses the geometric concepts involving non-right triangles, which may include the more specific NAEP concept.
3 Geometry	G. Geometry and Measurement	The NAEP standard is addressed by the SAT goal that also addresses measurement.
3.1 Dimension and shape		
3.1.c Give precise mathematical descriptions or definitions of geometric shapes in the plane and in three-dimensional space.	G.1 Points and lines in the plane (locus, parallel, perpendicular), including use of geometric notation (length, segments, lines, rays, and congruence); G.2 Angles in the plane – including complementary, supplementary, right, vertical, and parallel lines; G.3 Triangles (nonspecial) – area, perimeter, ratio and comparison of measures, triangle inequality, interior and exterior angles, similar triangles (similarity added for the PSAT/NMSQT); G.4 Special triangles (30-60-90, isosceles, equilateral, etc.) – including area, perimeter, ratio and comparison of measures, angles, Pythagorean theorem, similar triangles, and problems in which an alternate method of solution involves trigonometry; G.5 Circles – area, circumference, sectors, properties of tangent lines, polygons with circles; G.6 Polygons (without circles); G.7 Solid geometric figures – cubes, prisms, cylinders, cones, spheres, pyramids (volume, surface area, properties)	The NAEP objective specifically addresses descriptions and definitions of 2-D and 3-D shapes. The SAT objectives more broadly address the geometric concepts involving 1-D, 2-D, and 3-D shapes, which may include relationships as well as problem solving.
3.1.d Draw or sketch from a written description plane figures and planar images of three-dimensional figures.	G.6 Polygons (without circles); G.7 Solid geometric figures – cubes, prisms, cylinders, cones, spheres, pyramids (volume, surface area, properties); G.8 Geometric Perception	The NAEP objective specifically addresses drawing 2-D and 3-D shapes. The SAT objectives more broadly address the geometric concepts involving shapes, which may include properties, relationships, and problem solving.

NAEP Mathematics	SAT Mathematics	Similarities and Differences
3.1.e Use two-dimensional representations of three-dimensional objects to visualize and solve problems.	G.7 Solid geometric figures – cubes, prisms, cylinders, cones, spheres, pyramids (volume, surface area, properties); G.8 Geometric Perception	The NAEP objective specifically addresses visualizing 3-D shapes and solving problems. The SAT objectives more broadly address the geometric concepts involving 3-D shapes, which may include properties and relationships, and possibly problem solving.
3.1.f Analyze properties of three-dimensional figures including spheres and hemispheres.	G.7 Solid geometric figures – cubes, prisms, cylinders, cones, spheres, pyramids (volume, surface area, properties)	The NAEP objective specifically addresses the properties of 3-D shapes. The SAT objective more broadly addresses the geometric concepts involving 3-D shapes, including their measures as well as properties, and may include problem solving as well.
3.2 Transformation of shapes and preservation of properties		
3.2.a Recognize or identify types of symmetries (e.g., point, line, rotational, self-congruence) of two- and three-dimensional figures.	G.8 Geometric Perception	The NAEP objective specifically addresses recognizing symmetries of 2-D and 3-D shapes. The SAT objective more broadly addresses geometric concepts involving geometric perception, which likely include non-symmetrical shapes.
3.2.b Give or recognize the precise mathematical relationship (e.g., congruence, similarity, orientation) between a figure and its image under a transformation.	G.8 Geometric Perception	The NAEP objective specifically addresses recognizing types of transformations involving 2-D shapes. The SAT objective more broadly addresses geometric concepts involving geometric perception, which may include the more specific NAEP concept.
3.2.c Perform or describe the effect of a single transformation on two- and three-dimensional geometric shapes (reflections across lines of symmetry, rotations, translations, and dilations).	G.8 Geometric Perception	The NAEP objective specifically addresses the effect of a single transformation involving 2-D and 3-D shapes. The SAT objective more broadly addresses geometric concepts involving geometric perception, which may include the more specific NAEP concept.
3.2.d Identify transformations, combinations, or subdivisions of shapes that preserve the area of two-dimensional figures or the volume of three-dimensional figures.	G.5 Circles – area, circumference, sectors, properties of tangent lines, polygons with circles; G.6 Polygons (without circles); G.7 Solid geometric figures – cubes, prisms, cylinders, cones, spheres, pyramids (volume, surface area, properties); G.8 Geometric Perception	The NAEP objective specifically addresses combining and subdividing 2-D and 3-D shapes. The SAT objectives more broadly address geometric concepts involving 2-D and 3-D shapes, which may include the more specific NAEP concept.

NAEP Mathematics	SAT Mathematics	Similarities and Differences
3.2.e Justify relationships of congruence and similarity and apply these relationships using scaling and proportional reasoning.	G.3 Triangles (nonspecial) – area, perimeter, ratio and comparison of measures, triangle inequality, interior and exterior angles, similar triangles (similarity added for the PSAT/NMSQT); G.4 Special triangles (30-60-90, isosceles, equilateral, etc.) – including area, perimeter, ratio and comparison of measures, angles, Pythagorean theorem, similar triangles, and problems in which an alternate method of solution involves trigonometry	The NAEP objective specifically addresses relationships of congruence and similarity, but likely involving various types of 2-D and 3-D shapes. The SAT objectives address the concepts of congruence and similarity involving triangles.
3.2.g Perform or describe the effects of successive transformations.	G.8 Geometric Perception	The NAEP objective specifically addresses the effect of successive transformations on shapes. The SAT objective more broadly addresses geometric concepts involving geometric perception, which may include the more specific NAEP concept.
3.3 Relationships between geometric figures		
3.3.b Apply geometric properties and relationships to solve problems in two and three dimensions.	G.1 Points and lines in the plane (locus, parallel, perpendicular), including use of geometric notation (length, segments, lines, rays, and congruence); G.2 Angles in the plane – including complementary, supplementary, right, vertical, and parallel lines; G.3 Triangles (nonspecial) – area, perimeter, ratio and comparison of measures, triangle inequality, interior and exterior angles, similar triangles (similarity added for the PSAT/NMSQT); G.4 Special triangles (30-60-90, isosceles, equilateral, etc.) – including area, perimeter, ratio and comparison of measures, angles, Pythagorean theorem, similar triangles, and problems in which an alternate method of solution involves trigonometry; G.5 Circles – area, circumference, sectors, properties of tangent lines, polygons with circles; G.6 Polygons (without circles); G.7 Solid geometric figures – cubes, prisms, cylinders, cones, spheres, pyramids (volume, surface area, properties)	The NAEP objective and the SAT objectives are equivalent.

NAEP Mathematics	SAT Mathematics	Similarities and Differences
3.3.c Represent problem situations with geometric models to solve mathematical or real-world problems.	G.1 Points and lines in the plane (locus, parallel, perpendicular), including use of geometric notation (length, segments, lines, rays, and congruence); G.2 Angles in the plane – including complementary, supplementary, right, vertical, and parallel lines; G.3 Triangles (nonspecial) – area, perimeter, ratio and comparison of measures, triangle inequality, interior and exterior angles, similar triangles (similarity added for the PSAT/NMSQT); G.4 Special triangles (30-60-90, isosceles, equilateral, etc.) – including area, perimeter, ratio and comparison of measures, angles, Pythagorean theorem, similar triangles, and problems in which an alternate method of solution involves trigonometry; G.5 Circles – area, circumference, sectors, properties of tangent lines, polygons with circles; G.6 Polygons (without circles); G.7 Solid geometric figures – cubes, prisms, cylinders, cones, spheres, pyramids (volume, surface area, properties)	The NAEP objective addresses solving problems involving geometric models. The SAT objectives may include applications, but likely address the properties and relationships of geometric figures in a more general sense.
3.3.d Use the Pythagorean theorem to solve problems in two- or three-dimensional situations.	G.4 Special triangles (30-60-90, isosceles, equilateral, etc.) – including area, perimeter, ratio and comparison of measures, angles, Pythagorean theorem, similar triangles, and problems in which an alternate method of solution involves trigonometry	The NAEP objective specifically addresses applications of the Pythagorean theorem. The SAT objective more broadly addresses geometric concepts involving right triangles, which likely include the more specific NAEP concept.

NAEP Mathematics	SAT Mathematics	Similarities and Differences
3.3.e Recall and interpret definitions and basic properties of congruent and similar triangles, circles, quadrilaterals, polygons, parallel, perpendicular and intersecting lines, and associated angle relationships.	G.2 Angles in the plane – including complementary, supplementary, right, vertical, and parallel lines; G.3 Triangles (nonspecial) – area, perimeter, ratio and comparison of measures, triangle inequality, interior and exterior angles, similar triangles (similarity added for the PSAT/NMSQT); G.4 Special triangles (30-60-90, isosceles, equilateral, etc.) – including area, perimeter, ratio and comparison of measures, angles, Pythagorean theorem, similar triangles, and problems in which an alternate method of solution involves trigonometry; G.5 Circles – area, circumference, sectors, properties of tangent lines, polygons with circles; G.6 Polygons (without circles)	The NAEP objective and the SAT objectives are equivalent.
3.3.f Analyze properties or relationships of triangles, quadrilaterals, and other polygonal plane figures.	G.3 Triangles (nonspecial) – area, perimeter, ratio and comparison of measures, triangle inequality, interior and exterior angles, similar triangles (similarity added for the PSAT/NMSQT); G.4 Special triangles (30-60-90, isosceles, equilateral, etc.) – including area, perimeter, ratio and comparison of measures, angles, Pythagorean theorem, similar triangles, and problems in which an alternate method of solution involves trigonometry; G.5 Circles – area, circumference, sectors, properties of tangent lines, polygons with circles; G.6 Polygons (without circles)	The NAEP objective specifically addresses the analysis of 2-D shapes. The SAT objectives more broadly address geometric concepts involving 2-D shapes, which likely include analysis as well as knowledge and skills at a more basic level of understanding.
3.3.g Analyze properties and relationships of parallel, perpendicular, or intersecting lines including the angle relationships that arise in these cases.	G.1 Points and lines in the plane (locus, parallel, perpendicular), including use of geometric notation (length, segments, lines, rays, and congruence); G.2 Angles in the plane – including complementary, supplementary, right, vertical, and parallel lines	The NAEP objective specifically addresses the analysis of lines. The SAT objectives more broadly address geometric concepts involving lines and angles, which likely include analysis as well as knowledge and skills at a more basic level of understanding.

NAEP Mathematics	SAT Mathematics	Similarities and Differences
3.3.h Analyze properties of circles and the intersections of lines and circles (inscribed angles, central angles, tangents, secants, and chords).	G.5 Circles – area, circumference, sectors, properties of tangent lines, polygons with circles	The NAEP objective specifically addresses the analysis of circles. The SAT objective likely addresses the concepts involving circles at the level of analysis as well as other levels of the concepts at a more basic level of understanding.
3.4 Position, direction, and coordinate geometry		
3.4.a Solve problems involving the coordinate plane such as the distance between two points, the midpoint of a segment, or slopes of perpendicular or parallel lines.	G.1 Points and lines in the plane (locus, parallel, perpendicular), including use of geometric notation (length, segments, lines, rays, and congruence); G.9 Coordinate Geometry, including midpoint formula, distance formula, slope, and properties of parallel and perpendicular lines	The NAEP objective specifically addresses applications of concepts in the coordinate plane. The SAT objectives more broadly address geometric concepts involving the coordinate plane, which likely include the more specific NAEP concept.
3.4.b Describe the intersections of lines in the plane and in space, intersections of a line and a plane, or of two planes in space.	G.1 Points and lines in the plane (locus, parallel, perpendicular), including use of geometric notation (length, segments, lines, rays, and congruence); G.8 Geometric Perception	The NAEP objective and the SAT objectives are equivalent.
3.4.c Describe or identify conic sections and other cross-sections of solids.	G.8 Geometric Perception	The NAEP objective specifically addresses conic sections and cross-sections. The SAT objective more broadly addresses the geometric concepts involving visualization skills, which may include the more specific NAEP concept.
3.4.d Represent two-dimensional figures algebraically using coordinates and/or equations.	G.6 Polygons (without circles); G.9 Coordinate Geometry, including midpoint formula, distance formula, slope, and properties of parallel and perpendicular lines	The NAEP objective specifically addresses using algebra to describe 2-D shapes on a coordinate plane. The SAT objectives more broadly address the geometric concepts involving 2-D shapes and the coordinate plane, which may include the more specific NAEP objective.
3.4.e * Use vectors to represent velocity and direction; multiply a vector by a scalar and add vectors both algebraically and graphically.		No SAT objectives align to NAEP 3.4.e.
3.4.f Find an equation of a circle given its center and radius and, given an equation of a circle, find its center and radius.		No SAT objectives align to NAEP 3.4.f.

NAEP Mathematics	SAT Mathematics	Similarities and Differences
3.4.g * Graph ellipses and hyperbolas whose axes are parallel to the coordinate axes and demonstrate understanding of the relationship between their standard algebraic form and their graphical characteristics.		No SAT objectives align to NAEP 3.4.g.
3.4.h * Represent situations and solve problems involving polar coordinates.		No SAT objectives align to NAEP 3.4.h.
3.5 Mathematical reasoning in geometry		
3.5.a Make, test, and validate geometric conjectures using a variety of methods including deductive reasoning and counterexamples.		No SAT objectives align to NAEP 3.5.a.
3.5.b Determine the role of hypotheses, logical implications, and conclusion in proofs of geometric theorems.		No SAT objectives align to NAEP 3.5.b.
3.5.c Analyze or explain a geometric argument by contradiction.		No SAT objectives align to NAEP 3.5.c.
3.5.d Analyze or explain a geometric proof of the Pythagorean theorem.		No SAT objectives align to NAEP 3.5.d.
3.5.e Prove basic theorems about congruent and similar triangles and circles.		No SAT objectives align to NAEP 3.5.e.
4 Data analysis, statistics, and probability	D. Data Analysis, Statistics, and Probability	
4.1 Data representation		
4.1.a Read or interpret graphical or tabular representations of data.	D.1 Data Interpretation (tables, charts, line graphs, circle graphs, histograms, pictographs, scatterplots and matrices)	SAT D.1 is general in nature and is likely to describe one or more specific NAEP objectives within the Data Analysis, Statistics, and Probability standard, those involving data presentation.
4.1.b For a given set of data, complete a graph and solve a problem using the data in the graph (histograms, scatterplots, and line graphs).	D.1 Data Interpretation (tables, charts, line graphs, circle graphs, histograms, pictographs, scatterplots and matrices)	See NAEP 4.1.a.
4.1.c Solve problems involving univariate or bivariate data.	D.1 Data Interpretation (tables, charts, line graphs, circle graphs, histograms, pictographs, scatterplots and matrices)	See NAEP 4.1.a.
4.1.d Given a graphical or tabular representation of a set of data, determine whether information is represented effectively and appropriately.	D.1 Data Interpretation (tables, charts, line graphs, circle graphs, histograms, pictographs, scatterplots and matrices)	See NAEP 4.1.a.

NAEP Mathematics	SAT Mathematics	Similarities and Differences
4.1.e Compare and contrast different graphical representations of univariate and bivariate data.	D.1 Data Interpretation (tables, charts, line graphs, circle graphs, histograms, pictographs, scatterplots and matrices)	See NAEP 4.1.a.
4.1.f Organize and display data in a spreadsheet in order to recognize patterns and solve problems.	D.1 Data Interpretation (tables, charts, line graphs, circle graphs, histograms, pictographs, scatterplots and matrices)	See NAEP 4.1.a.
4.2 Characteristics of data sets		
4.2.a Calculate, interpret, or use summary statistics for distributions of data including measures of typical value (mean, median), position (quartiles, percentiles), and spread (range, interquartile range, variance, and standard deviation).	D.2 Statistics – measures of central tendency (mean, median, mode) - (median and mode added for PSAT/NMSQT)	SAT D.2 is general in nature and is likely to describe one or more specific NAEP objectives within the Data Analysis, Statistics, and Probability standard, those involving the characteristics of data sets.
4.2.b Recognize how linear transformations of one-variable data affect mean, median, mode, range, interquartile range, and standard deviation.	D.2 Statistics – measures of central tendency (mean, median, mode) - (median and mode added for PSAT/NMSQT)	See NAEP 4.2.a.
4.2.c Determine the effect of outliers on mean, median, mode, range, interquartile range, or standard deviation.	D.2 Statistics – measures of central tendency (mean, median, mode) - (median and mode added for PSAT/NMSQT)	See NAEP 4.2.a.
4.2.d Compare data sets using summary statistics (mean, median, mode, range, interquartile range, or standard deviation) describing the same characteristic for two different populations or subsets of the same population.	D.2 Statistics – measures of central tendency (mean, median, mode) - (median and mode added for PSAT/NMSQT)	See NAEP 4.2.a.
4.2.e Approximate a trend line if a linear pattern is apparent in a scatterplot or use a graphing calculator to determine a least-squares regression line and use the line or equation to make predictions.	D.1 Data Interpretation (tables, charts, line graphs, circle graphs, histograms, pictographs, scatterplots and matrices)	See NAEP 4.1.a.
4.2.f Recognize that the correlation coefficient is a number from -1 to +1 that measures the strength of the linear relationship between two variables; visually estimate the correlation coefficient (e.g., positive or negative, closer to 0, .5, or 1.0) of a scatterplot.	D.1 Data Interpretation (tables, charts, line graphs, circle graphs, histograms, pictographs, scatterplots and matrices)	See NAEP 4.1.a.

NAEP Mathematics	SAT Mathematics	Similarities and Differences
4.2.g Know and interpret the key characteristics of a normal distribution such as shape, center (mean), and spread (standard deviation).		No SAT objectives align to NAEP 4.2.g.
4.3 Experiments and samples		
4.3.a Identify possible sources of bias in sample surveys and describe how such bias can be controlled and reduced.		No SAT objectives align to NAEP 4.3.a.
4.3.b Recognize and describe a method to select a simple random sample.		No SAT objectives align to NAEP 4.3.b.
4.3.c * Draw inferences from samples, such as estimates of proportions in a population, estimates of population means, or decisions about differences in means for two "treatments."		No SAT objectives align to NAEP 4.3.c.
4.3.d Identify or evaluate the characteristics of a good survey or of a well-designed experiment.		No SAT objectives align to NAEP 4.3.d.
4.3.e * Recognize the differences in design and in conclusions between randomized experiments and observational studies.		No SAT objectives align to NAEP 4.3.e.
4.4 Probability		
4.4.a Recognize whether two events are independent or dependent.	D.3 Probability – elementary, geometric	SAT D.3 is general in nature and is likely to describe one or more specific NAEP objectives within the Data Analysis, Statistics, and Probability standard, those involving probability.
4.4.b Determine the theoretical probability of simple and compound events in familiar or unfamiliar contexts.	D.3 Probability – elementary, geometric	See NAEP 4.4.a.
4.4.c Given the results of an experiment or simulation, estimate the probability of simple or compound events in familiar or unfamiliar contexts.	D.3 Probability – elementary, geometric	See NAEP 4.4.a.
4.4.d Use theoretical probability to evaluate or predict experimental outcomes.	D.3 Probability – elementary, geometric	See NAEP 4.4.a.
4.4.e Determine the number of ways an event can occur using tree diagrams, formulas for combinations and permutations, or other counting techniques.	N.6 Counting problems (permutations, combinations, multiplication principal); D.3 Probability – elementary, geometric	The NAEP objective and the SAT objectives are equivalent.

NAEP Mathematics	SAT Mathematics	Similarities and Differences
4.4.h Determine the probability of independent and dependent events.	D.3 Probability – elementary, geometric	See NAEP 4.4.a.
4.4.i Determine conditional probability using two-way tables.	D.3 Probability – elementary, geometric	See NAEP 4.4.a.
4.4.j Interpret and apply probability concepts to practical situations.	D.3 Probability – elementary, geometric	See NAEP 4.4.a.
4.4.k *Use the binomial theorem to solve problems.		No SAT objectives align to NAEP 4.4.k.
4.5 Mathematical reasoning with data		
4.5.a Identify misleading uses of data in real-world settings and critique different ways of presenting and using information.	D.1 Data Interpretation (tables, charts, line graphs, circle graphs, histograms, pictographs, scatterplots and matrices); D.2 Statistics – measures of central tendency (mean, median, mode) - (median and mode added for PSAT/NMSQT)	The NAEP objective specifically addresses the misuse of data representations. The SAT objectives more broadly address the concepts involving data interpretation, which may include the more specific NAEP objective.
4.5.b Distinguish relevant from irrelevant information, identify missing information, and either find what is needed or make appropriate approximations.		No SAT objectives align to NAEP 4.5.b.
4.5.c *Recognize, use, and distinguish between the processes of mathematical (deterministic) and statistical modeling.		No SAT objectives align to NAEP 4.5.c.
4.5.d Recognize when arguments based on data confuse correlation with causation.		No SAT objectives align to NAEP 4.5.d.
4.5.e * Recognize and explain the potential errors caused by extrapolating from data.		No SAT objectives align to NAEP 4.5.e.
5 Algebra	A. Algebra and Functions	
5.1 Patterns, relations, and functions		
5.1.a Recognize, describe, or extend numerical patterns, including arithmetic and geometric progressions.	N.4 Sequences and series (rule given explicitly) – finding specific term or sum of terms, exponential growth	The NAEP objective broadly addresses patterning. The SAT objective more specifically addresses the concepts involving sequences and series, which are addressed by the NAEP objective.

NAEP Mathematics	SAT Mathematics	Similarities and Differences
5.1.b Express linear and exponential functions in recursive and explicit form given a table, verbal description, or some terms of a sequence.	A.2 Algebraic representations, translation, and algebraic word problems (those that usually require an algebraic equation to solve); A.3 Linear equations and inequalities; A.10 Basic concepts of algebraic functions: Using new definitions as operations; Function notation and evaluation; Functions as models; Qualitative behavior of functions; Coordinate – linear equations; Coordinate – quadratic equations; Concepts of domain and range; Transformations	The NAEP objective specifically addresses recursive functions. The SAT objectives more broadly address the concepts involving equations and functions, which may include the more specific NAEP objective.
5.1.e Identify or analyze distinguishing properties of linear, quadratic, rational, exponential, or *trigonometric functions from tables, graphs, or equations.	A.3 Linear equations and inequalities; A.5 Rational equations and inequalities; A.10 Basic concepts of algebraic functions: Using new definitions as operations; Function notation and evaluation; Functions as models; Qualitative behavior of functions; Coordinate – linear equations; Coordinate – quadratic equations; Concepts of domain and range; Transformations	The NAEP objective specifically addresses properties of functions. The SAT objectives more broadly address those same concepts, and likely include the more specific NAEP objective.
5.1.g Determine whether a relation, given in verbal, symbolic, tabular, or graphical form, is a function.	A.10 Basic concepts of algebraic functions: Using new definitions as operations; Function notation and evaluation; Functions as models; Qualitative behavior of functions; Coordinate – linear equations; Coordinate – quadratic equations; Concepts of domain and range; Transformations	The NAEP objective specifically addresses identifying a function. The SAT objective more broadly addresses the concepts involving equations and functions, which likely include the more specific NAEP objective.
5.1.h Recognize and analyze the general forms of linear, quadratic, rational, exponential, or *trigonometric functions.	A.3 Linear equations and inequalities; A.5 Rational equations and inequalities; A.10 Basic concepts of algebraic functions: Using new definitions as operations; Function notation and evaluation; Functions as models; Qualitative behavior of functions; Coordinate – linear equations; Coordinate – quadratic equations; Concepts of domain and range; Transformations	The NAEP objective specifically addresses the basic concepts of functions, including trigonometric functions. The SAT objectives more broadly address those same concepts, and likely include the more specific NAEP objective, except for trigonometric functions.

NAEP Mathematics	SAT Mathematics	Similarities and Differences
5.1.i Determine the domain and range of functions given in various forms and contexts.	A.10 Basic concepts of algebraic functions: Using new definitions as operations; Function notation and evaluation; Functions as models; Qualitative behavior of functions; Coordinate – linear equations; Coordinate – quadratic equations; Concepts of domain and range; Transformations	The NAEP objective specifically addresses the domain and range of functions. The SAT objective more broadly addresses the concepts involving equations and functions, which likely include the more specific NAEP objective.
5.1.j * Given a function, determine its inverse if it exists and explain the contextual meaning of the inverse for a given situation.	A.10 Basic concepts of algebraic functions: Using new definitions as operations; Function notation and evaluation; Functions as models; Qualitative behavior of functions; Coordinate – linear equations; Coordinate – quadratic equations; Concepts of domain and range; Transformations	The NAEP objective specifically addresses the inverse of a function. The SAT objective more broadly addresses the concepts involving equations and functions, which may include the more specific NAEP objective.
5.2 Algebraic representations		
5.2.a Create and translate between different representations of algebraic expressions, equations, and inequalities (e.g., linear, quadratic, exponential, or *trigonometric) using symbols, graphs, tables, diagrams, or written descriptions.	A.2 Algebraic representations, translation, and algebraic word problems (those that usually require an algebraic equation to solve); A.3 Linear equations and inequalities; A.5 Rational equations and inequalities; A.7 Quadratics – factoring difference of two squares, perfect square trinomials, and $ax^2 + bx + c$ trinomials (that are factorable), solving factorable quadratics; A.9 Direct and inverse variation	The NAEP objective broadly addresses the basic concepts of algebraic representations, including trigonometric representations. The SAT objectives address those same concepts, and likely include the NAEP objective, except for trigonometric functions.
5.2.b Analyze or interpret relationships expressed in symbols, graphs, tables, diagrams (including Venn diagrams), or written descriptions and evaluate the relative advantages or disadvantages of different representations to answer specific questions.	A.2 Algebraic representations, translation, and algebraic word problems (those that usually require an algebraic equation to solve); A.9 Direct and inverse variation	The NAEP objective addresses the concepts of analyzing, interpreting, and evaluating algebraic representations. The SAT objectives address those concepts at a more basic level.
5.2.d Perform or interpret transformations on the graphs of linear, quadratic, exponential, and *trigonometric functions.	A.3 Linear equations and inequalities; A.10 Basic concepts of algebraic functions: Using new definitions as operations; Function notation and evaluation; Functions as models; Qualitative behavior of functions; Coordinate – linear equations; Coordinate – quadratic equations; Concepts of domain and range; Transformations	The NAEP objective addresses the concepts of transformations of algebraic functions. The SAT objectives address those same concepts, and likely include the NAEP objective, except for trigonometric functions.

NAEP Mathematics	SAT Mathematics	Similarities and Differences
5.2.e Make inferences or predictions using an algebraic model of a situation.	A.3 Linear equations and inequalities; A.4 Systems of linear equations and inequalities; A.9 Direct and inverse variation	The NAEP objective specifically addresses the concepts of algebraic modeling and predictions. The SAT objectives more broadly address the concepts involving equations, which may include the more specific NAEP objective.
5.2.f Given a real-world situation, determine if a linear, quadratic, rational, exponential, logarithmic, or *trigonometric function fits the situation.	A.2 Algebraic representations, translation, and algebraic word problems (those that usually require an algebraic equation to solve); A.3 Linear equations and inequalities; A.5 Rational equations and inequalities; A.7 Quadratics – factoring difference of two squares, perfect square trinomials, and $ax^2 + bx + c$ trinomials (that are factorable), solving factorable quadratics; A.10 Basic concepts of algebraic functions: Using new definitions as operations; Function notation and evaluation; Functions as models; Qualitative behavior of functions; Coordinate – linear equations; Coordinate – quadratic equations; Concepts of domain and range; Transformations	The NAEP objective specifically addresses the concepts of algebraic modeling and predictions. The SAT objectives more broadly address the concepts involving equations and functions, which may include the more specific NAEP objective.
5.2.g Solve problems involving exponential growth and decay.	A.2 Algebraic representations, translation, and algebraic word problems (those that usually require an algebraic equation to solve)	The NAEP objective specifically addresses applications involving exponential models. The SAT objective more broadly addresses the concepts involving algebraic word problems, which may include the more specific NAEP objective.
5.2.h * Analyze properties of exponential, logarithmic, and rational functions.	A.5 Rational equations and inequalities; A.10 Basic concepts of algebraic functions: Using new definitions as operations; Function notation and evaluation; Functions as models; Qualitative behavior of functions; Coordinate – linear equations; Coordinate – quadratic equations; Concepts of domain and range; Transformations	The NAEP objective specifically addresses the properties of exponential, logarithmic, and rational functions. The SAT objectives more broadly address the concepts involving equations and functions, which may include the more specific NAEP objective.

NAEP Mathematics	SAT Mathematics	Similarities and Differences
5.3 Variables, expressions, and operations		
5.3.b Write algebraic expressions, equations, or inequalities to represent a situation.	A.2 Algebraic representations, translation, and algebraic word problems (those that usually require an algebraic equation to solve); A.3 Linear equations and inequalities; A.4 Systems of linear equations and inequalities; A.5 Rational equations and inequalities; A.9 Direct and inverse variation	The NAEP objective broadly addresses the basic concepts of representing a situation algebraically. The SAT objectives more broadly address the concepts involving equations and inequalities, which may include the more specific NAEP objective.
5.3.c Perform basic operations, using appropriate tools, on algebraic expressions including polynomial and rational expressions.	A.5 Rational equations and inequalities	The NAEP objective broadly addresses the basic concepts of algebraic expressions. The SAT objective addresses rational equations and inequalities, which may include the NAEP objective, as it involves rational expressions, but not for polynomials.
5.3.d Write equivalent forms of algebraic expressions, equations, or inequalities to represent and explain mathematical relationships.	A.2 Algebraic representations, translation, and algebraic word problems (those that usually require an algebraic equation to solve); A.3 Linear equations and inequalities; A.4 Systems of linear equations and inequalities; A.5 Rational equations and inequalities	The NAEP objective and the SAT objectives are equivalent.
5.3.e Evaluate algebraic expressions including polynomials and rational expressions.	A.5 Rational equations and inequalities	The NAEP objective broadly addresses evaluating algebraic expressions. The SAT objective addresses rational equations and inequalities, which may include the NAEP objective, as it involves rational expressions, but not for polynomials.
5.3.f Use function notation to evaluate a function at a specified point in its domain and combine functions by addition, subtraction, multiplication, division, and composition.	A.10 Basic concepts of algebraic functions: Using new definitions as operations; Function notation and evaluation; Functions as models; Qualitative behavior of functions; Coordinate – linear equations; Coordinate – quadratic equations; Concepts of domain and range; Transformations	The NAEP objective specifically addresses evaluating a function for a specific value and performing operations on functions. The SAT objective more broadly address the concepts involving functions, which include evaluating a function and may include combining functions using operations.
5.3.g * Determine the sum of finite and infinite arithmetic and geometric series.	N.4 Sequences and series (rule given explicitly) – finding specific term or sum of terms, exponential growth	The NAEP objective specifically addresses finding the sums of series. The SAT objective more broadly addresses the concepts involving sequences and series, which likely include the more specific NAEP objective.

NAEP Mathematics	SAT Mathematics	Similarities and Differences
5.3.h Use basic properties of exponents and *logarithms to solve problems.	A.8 Manipulation with integer and rational exponents and using rules for exponents	The NAEP objective specifically addresses applications of exponents and logarithms. The SAT objective more broadly addresses the concepts involving the basic concepts of exponents, which may include the more specific NAEP objective, except for logarithms.
5.4 Equations and inequalities	A.6 Radical equations	The SAT objective is not specifically addressed by any NAEP objectives, but is related to the NAEP goal.
5.4.a Solve linear, rational, or quadratic equations or inequalities, including those involving absolute value.	A.3 Linear equations and inequalities; A.5 Rational equations and inequalities; A.7 Quadratics – factoring difference of two squares, perfect square trinomials, and $ax^2 + bx + c$ trinomials (that are factorable), solving factorable quadratics	The NAEP objective specifically addresses solving equations and inequalities. The SAT objectives more broadly address the concepts of equations and inequalities, but likely include the more specific NAEP objective.
5.4.c Analyze situations, develop mathematical models, or solve problems using linear, quadratic, exponential, or logarithmic equations or inequalities symbolically or graphically.	A.2 Algebraic representations, translation, and algebraic word problems (those that usually require an algebraic equation to solve); A.3 Linear equations and inequalities; A.4 Systems of linear equations and inequalities	The NAEP objective specifically addresses the application of equations and inequalities in solving real-world problems. The SAT objectives more broadly address the concepts of equations and inequalities, but likely include the more specific NAEP objective.
5.4.d Solve (symbolically or graphically) a system of equations or inequalities and recognize the relationship between the analytical solution and graphical solution.	N.1 Properties of integers (odd/even, primes, factors, GCF, LCM, properties of 0 and 1, positive/negative, place value), elementary number theory – modular arithmetic, Diophantine equations, divisibility, remainders; A.4 Systems of linear equations and inequalities	The NAEP objective specifically addresses solving systems of equations and inequalities. The SAT objectives more broadly address the concepts of systems of equations and inequalities, but likely include the more specific NAEP objective. SAT N.1 addresses Diophantine equations, which are interpreted to mean a subset of systems of equations.
5.4.e Solve problems involving special formulas such as: $A = P(I + r)^t$, $A = Pe^{rt}$.		No SAT objectives align to NAEP 5.4.e.
5.4.f Solve an equation or formula involving several variables for one variable in terms of the others.		No SAT objectives align to NAEP 5.4.f.
5.4.g Solve quadratic equations with complex roots.		No SAT objectives align to NAEP 5.4.g.

NAEP Mathematics	SAT Mathematics	Similarities and Differences
5.5 Mathematical reasoning in algebra		
5.5.a Use algebraic properties to develop a valid mathematical argument.	N.7 Logic/logical reasoning	The NAEP objective specifically addresses using algebraic reasoning to develop an argument. The SAT objective more broadly addresses the concepts of logical reasoning, which may include the more specific NAEP objective.
5.5.b Determine the role of hypotheses, logical implications, and conclusions in algebraic argument.	N.7 Logic/logical reasoning	The NAEP objective specifically addresses analyzing algebraic reasoning. The SAT objective more broadly addresses the concepts of logical reasoning, which may include the more specific NAEP objective.
5.5.c Explain the use of relational conjunctions (and, or) in algebraic arguments.	N.7 Logic/logical reasoning	The NAEP objective specifically addresses the use of "and", "or" in algebraic reasoning. The SAT objective more broadly addresses the concepts of logical reasoning, which may include the more specific NAEP objective.

Some of the grade 12 objectives are marked with an "*." This denotes objectives that describe mathematics content beyond that typically taught in a standard 3-year course of study (the equivalent of 1 year of geometry and 2 years of algebra). Therefore, these objectives will be selected less often than the others for inclusion on the assessments. Although all test items will be assigned a primary classification, some test items could potentially fall into more than one content area or under more than one objective.

Appendix B: NAEP Objectives Not Addressed in the SAT Specifications

The 38 NAEP objectives not addressed in the SAT specifications are listed below.

1.1.g Represent, interpret, or compare expressions or problem situations involving absolute values.

1.2.b Identify situations where estimation is appropriate, determine the needed degree of accuracy, and analyze* the effect of the estimation method on the accuracy of results.

1.2.c Verify solutions or determine the reasonableness of results in a variety of situations.

1.2.d Estimate square or cube roots of numbers less than 1,000 between two whole numbers.

1.3.c Perform arithmetic operations with expressions involving absolute value.

1.5.e Apply basic properties of operations, including conventions about the order of operations.

2.1.i Solve problems involving rates such as speed, density, population density, or flow rates.

2.2.a Recognize that geometric measurements (length, area, perimeter, and volume) depend on the choice of a unit, and apply such units in expressions, equations, and problem solutions.

2.2.b Solve problems involving conversions within or between measurement systems, given the relationship between the units.

2.2.d Understand that numerical values associated with measurements of physical quantities are approximate, are subject to variation, and must be assigned units of measurement.

2.2.e Determine appropriate accuracy of measurement in problem situations (e.g., the accuracy of measurement of the dimensions to obtain a specified accuracy of area) and find the measure to that degree of accuracy.

2.2.f Construct or solve problems involving scale drawings.

2.3.d Interpret and use the identity $\sin^2 \theta + \cos^2 \theta = 1$ for angles θ between 0° and 90°; recognize this identity as a special representation of the Pythagorean theorem.

2.3.e *Determine the radian measure of an angle and explain how radian measurement is related to a circle of radius 1.

2.3.f *Use trigonometric formulas such as addition and double angle formulas.

3.4.e *Use vectors to represent velocity and direction; multiply a vector by a scalar and add vectors both algebraically and graphically.

3.4.f Find an equation of a circle given its center and radius and, given an equation of a circle, find its center and radius.

3.4.g *Graph ellipses and hyperbolas whose axes are parallel to the coordinate axes and demonstrate understanding of the relationship between their standard algebraic form and their graphical characteristics.

3.4.h *Represent situations and solve problems involving polar coordinates.

3.5.a Make, test, and validate geometric conjectures using a variety of methods including deductive reasoning and counterexamples.

3.5.b Determine the role of hypotheses, logical implications, and conclusion in proofs of geometric theorems.

3.5.c Analyze or explain a geometric argument by contradiction.

3.5.d Analyze or explain a geometric proof of the Pythagorean theorem.

3.5.e Prove basic theorems about congruent and similar triangles and circles.

4.2.g Know and interpret the key characteristics of a normal distribution such as shape, center (mean), and spread (standard deviation).

4.3.a Identify possible sources of bias in sample surveys and describe how such bias can be controlled and reduced.

4.3.b Recognize and describe a method to select a simple random sample.

4.3.c *Draw inferences from samples, such as estimates of proportions in a population, estimates of population means, or decisions about differences in means for two "treatments."

4.3.d Identify or evaluate the characteristics of a good survey or of a well-designed experiment.

4.3.e *Recognize the differences in design and in conclusions between randomized experiments and observational studies.

4.4.k *Use the binomial theorem to solve problems.

4.5.b Distinguish relevant from irrelevant information, identify missing information, and either find what is needed or make appropriate approximations.

4.5.c *Recognize, use, and distinguish between the processes of mathematical (deterministic) and statistical modeling.

4.5.d Recognize when arguments based on data confuse correlation with causation.

4.5.e *Recognize and explain the potential errors caused by extrapolating from data.

5.4.e Solve problems involving special formulas such as: $A = P(I + r)^t$, $A = Pe^{rt}$.

5.4.f Solve an equation or formula involving several variables for one variable in terms of the others.

5.4.g Solve quadratic equations with complex roots.

Appendix C: Decision Rules Applied in Operational Study

NAEP MATHEMATICS FRAMEWORK FOR ALIGNMENT: DECISION RULES

1) The objectives within the Algebra standard will be interpreted as aligning primarily to items containing one or more variables and not items containing only numerical expressions.
2) The objectives within the Number Operations and Properties standard will be interpreted as aligning primarily to numerical items; however, consideration is also to be given to items containing one or more variables.
3) The primary intent of objectives containing wording similar to the following is to assess mathematics in situations involving either real-world problem solving or problem solving in a mathematical context.
    - 1.1.g Represent, interpret, or compare expressions or problem situations involving absolute values.
    - 1.3.f Solve application problems involving numbers, including rational and common irrationals.
    - 1.4.c Use proportions to solve problems (including rates of change).
4) Some objectives contain multiple parts separated by the word "and" (see 1.5.f below). The intent of the objective may or may not be to assess all parts. If an item addresses only one part of the objective, panelists are asked to look for an alternative primary code. If an alternative code is not available, panelists are to note in the WAT that the item does not assess the entire objective.
    - 1.5.f Recognize properties of the number system (whole numbers, integers, rational numbers, real numbers, and complex numbers) and how they are related to each other, and identify examples of each type of number.
5) An objective that addresses expressions may also be aligned with an item containing an equation if symbolic manipulation across the equal sign is not required to answer the question.

SAT MATHEMATICS SPECIFICATIONS FOR ALIGNMENT: DECISION RULES

1) If an SAT goal/objective includes a parenthetical statement, panelists will consider the wording to be a clarification, rather than an exclusive list.
2) As with the NAEP objectives, the primary intent of SAT objectives containing wording similar to the following is to assess mathematics in situations involving either real-world problem solving or problem solving in a mathematical context.
    - N.3 Arithmetic word problems (those word problems that usually do not require an algebraic equation to solve, including word problems that involve percents and ratios)

- A.2 Algebraic representations, translation, and algebraic word problems (those that usually require an algebraic equation to solve)