National Assessment Governing Board

Content Alignment Studies of the 2009 National Assessment of Educational Progress for Grade 12 Reading and Mathematics with SAT and ACCUPLACER Assessments of these Subjects

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Interim Report: Comparative Analysis of the Test Blueprints and Specifications for 2009 NAEP Grade 12 Mathematics and ACCUPLACER Mathematics

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Introduction

WestEd has been contracted by the National Assessment Governing Board to study the extent to which the grade 12 National Assessment of Educational Progress (NAEP) is aligned in content and complexity to SAT and ACCUPLACER in reading and in mathematics. This project is part of the Governing Board's 12th grade preparedness initiative and will yield information on using the grade 12 NAEP to report on student preparedness for postsecondary activities. The overall study includes the examination of the alignment between the grade 12 NAEP in reading and mathematics and the SAT and ACCUPLACER assessments in those subjects.

As described in the study design document, the full study includes weeklong alignment coding sessions with replicate panels of content experts in reading and mathematics using the Web Alignment Tool. One component of the alignment study is an initial comparative analysis between the test blueprints and specifications for NAEP and each assessment, which occurs prior to the alignment coding sessions in order to inform advance expectations for alignment, as well as to raise potential alignment issues prior to item coding. The comparative analysis also provides an additional view of the alignment between the two assessments, which can be used in interpreting the item alignment analysis results, once completed. A future interim report will compare the mathematics framework for NAEP with the mathematics specifications for SAT, while interim reports comparing the reading framework for NAEP with the reading specifications for SAT and ACCUPLACER have been submitted to the Governing Board.

The results of this analysis are intended to provide information about the similarities and differences in assessment design and administration between the 2009 NAEP grade 12 Mathematics Assessment and the ACCUPLACER Mathematics Test (hereafter NAEP and ACCUPLACER, respectively). WestEd's lead content facilitator for mathematics conducted this analysis. For NAEP, the *Mathematics Framework for the 2009 National Assessment of Educational Progress* (National Assessment Governing Board, 2008) and *Mathematics Assessment and Item Specifications for the NAEP 2009 Mathematics Assessment* (National Assessment Governing Board, 2007) were used in this analysis. The content and numbering of the NAEP objectives for use in item coding are taken from the *Mathematics Framework for the 2009 National Assessment of Educational Progress*, Exhibits 3–7; formatting has been added for use in the NAEP–ACCUPLACER alignment study. The resulting format provides the NAEP objective numbering used in this report, and appears in the objective-level comparison table located in Appendix A. For ACCUPLACER, the College Board provided test specifications for three placement tests (Arithmetic, Elementary Algebra, and College Level Math), with expanded text for the alignment study to clarify the intent of the specifications with regard to the knowledge and skill required by the objectives under each goal. Following review of the specifications and discussions between WestEd and the Governing Board, it was decided that the specifications of the three placement tests should be combined into one set of content specifications for review in the study, for two reasons. First, there is an apparent progression of content across the three placement tests that, when taken together, is more comparable in scope to the NAEP framework than any single test would be. Second, aligning to the combined specifications would be more efficient in that only one NAEP–ACCUPLACER study would be required, rather than three separate studies, which would have been both time- and cost-

prohibitive. The resulting format combines test specifications and expanded descriptive detail provided by the College Board, with WestEd providing the objective numbering used in this report. [1]

This document presents the results of the comparative analysis. To provide context for the analysis, the report begins with an overview of each assessment's Purpose and Use, Test Administration Procedures, and Resources Available to Students. The subsequent sections include a detailed discussion of the findings of the content comparison in terms of Content Organization, Specificity of Content, and Student Performance. Later sections discuss the Number, Proportion, and Format of Items; Scoring Rubrics and Rules for Constructed-Response Items; and Grade Level Targeted by Items. The final section provides a Summary of Content Overlap and Implications for Item Alignment. The complete side-by-side comparison charts on which the analysis is based are provided as Appendix A. A list of the NAEP objectives not addressed in the ACCUPLACER specifications is provided as Appendix B. The final decision rules applied in the study are provided as Appendix C.

Purpose and Use

NAEP and ACCUPLACER are designed to measure the mathematics achievement of students at largely similar ages and grade levels. NAEP is administered to students in the 12th grade; ACCUPLACER is administered to students who are entering or planning to enter college at the freshman level. This includes students who are currently in 12th grade as well as students who have recently graduated from 12th grade. ACCUPLACER is also used for older students entering or re-entering college after a gap of time (College Board, 2009a).

Although both assessments measure the mathematics skills of students at similar ages and stages of academic progress, they serve different purposes for different audiences. NAEP, commonly referred to as "the Nation's Report Card," is administered to "representative samples" (National Center for Educational Statistics) of students across the country, and does not provide results for individual students, but rather for groups and subgroups of student populations. The information yielded by NAEP is intended to "hel[p] the public, educators, and policymakers understand strengths and weaknesses in student performance and make informed decisions about education" (National Assessment Governing Board, 2009, p. v). ACCUPLACER is primarily used by colleges to help determine the appropriate placement of incoming freshman students in college-level courses and "to determine if developmental classes would be beneficial before the students take college-level work" (College Board, 2009a). Therefore, ACCUPLACER provides results measuring the mathematics skills of individual students.

Test Administration Procedures

NAEP is a paper-based assessment administered to "random samples of students designed to be representative of the nation, different regions of the country, participating states, and large urban districts" (National Assessment Governing Board, 2009, p. 2). The items on the NAEP mathematics test are distributed across multiple test booklets "using a matrix sampling design" (p. 3), ensuring variability in booklet and item distribution among students. Students are given 50

[1] ACCUPLACER specifications information included in this report is the property of the College Board.

minutes to complete the NAEP mathematics assessment (National Assessment Governing Board, 2007, p. 99).

The NAEP mathematics test framework indicates that accommodations for students with special needs routinely provided by schools for their own testing programs are allowed in NAEP. Customary accommodations include:

- One-on-one testing
- Small group testing
- Extended time
- Oral reading of directions
- Large-print booklets
- Bilingual English/Spanish booklets
- Use of an aide to transcribe responses (National Assessment Governing Board, 2008, p. 67)

ACCUPLACER is computer adaptive and designed to be administered online. The test engine selects items for individual test-takers based on their responses to previous questions. Questions, therefore, must be answered in the order they are administered. The test is untimed.

The College Board makes a variety of presentation, responding, timing/scheduling, and setting accommodations available to eligible students taking College Board assessments. For students with disabilities, these include the following:

- Recorded tests
- Brailled versions of the tests
- Large print versions of the tests
- Calculators
- Interpreters, qualified readers or transcribers
- Screen display enlargement
- Other effective methods of making orally delivered materials available to individuals with hearing impairments (College Board, 2010, p. 67)

Examinees are also directed to consult with their ACCUPLACER testing center for information on the specific accommodations available to those with documented disabilities.

Resources Available to Students

The grade 12 NAEP mathematics assessment permits the limited use of certain approved tools. These tools include a spinner, folding card, ruler/protractor, and scientific calculator, all of which are provided to students for use during the test. Students are also permitted to bring and make restricted use of their own calculators, scientific or otherwise.[2] NAEP does not regulate the types of calculators students are allowed to use on the grade 12 test (National Assessment Governing Board, 2007, p. 100).

Calculator use is limited to "calculator blocks," sets of items that might prove difficult to solve without a calculator and for which calculator use has been approved. Approximately two-thirds of the blocks measure students' mathematical knowledge and skills without access to a calculator; the other third of the blocks allow the use of a calculator.

The intention of the Governing Board is that no items on the NAEP are designed to provide an advantage to students with graphing calculators. Estimated time required for any item should be based on the assumption that students are not using graphing calculators.

For ACCUPLACER, some items on the Arithmetic and Elementary Algebra tests allow students to use four-function or scientific (non-graphing) calculators. For these items, according to College Board, calculator use will not help students prove mastery of an assessed skill; it may, however, help students complete aspects of a mathematics problem (College Board, 2010, p. 64). When administered online, calculator items are identified with a calculator graphic, which becomes active for allowed items. On the paper companion forms used in the alignment study, there are no calculator items on the Arithmetic or Elementary Algebra tests. On the College Level Math test, a four-function or scientific calculator is allowed on all items (p. 65).[3]

Content Organization of NAEP and ACCUPLACER

The content specifications of NAEP and ACCUPLACER are organized hierarchically, in three levels of increasing specificity. For this study—for clarity of discussion, and for consistency with the nomenclature used in the Web Alignment Tool—the levels of both tests' specifications, from broadest to most specific, will be referred to as standards, goals, and objectives, respectively.

The NAEP framework is organized into five standards: Number Properties and Operations; Measurement; Geometry; Data Analysis, Statistics, and Probability; and Algebra.[4] These standards, however, are not discrete or "fragmented" sets. While the structure of these standards helps classify the mathematical content of the NAEP assessment, it is expected "that the objectives and test items built on them will, in many cases, cross content area boundaries" (National Assessment Governing Board, 2008, p. 7).

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[2] The intent of restrictions on calculator use at grade 12 is (1) to help ensure that items in calculator blocks cannot be solved in ways that are inconsistent with the knowledge and skills the items are intended to measure and (2) to maintain the security of NAEP test materials. These restrictions address issues such as calculators with QWERTY keyboards, communication between students during testing, and the use of stored formulas, algorithms, and other procedures (National Assessment Governing Board, 2007, pp. 100-101).

[3] Additionally, examinees are directed to consult with schools and test centers about specific site regulations regarding calculators, textbooks, protractors, notebooks, dictionaries, or other papers of any kind (College Board, 2009b).

[4] According to the NAEP test blueprint, Measurement and Geometry are collapsed together to comprise 30% of the complete assessment.

NAEP standards and goals are listed below. Counts of objectives for each standard are listed, and the full text of the 130 objectives is included in Appendix A.

Number Properties and Operations (20 objectives)

- 1.1 Number sense
- 1.2 Estimation
- 1.3 Number operations
- 1.4 Ratios and proportional reasoning
- 1.5 Properties of number and operations
- 1.6 Mathematical reasoning using number

Measurement (18 objectives)

- 2.1 Measuring physical attributes
- 2.2 Systems of measurement
- 2.3 Measurement in triangles

Geometry (30 objectives)

- 3.1 Dimension and shape
- 3.2 Transformation of shapes and preservation of properties
- 3.3 Relationships between geometric figures
- 3.4 Position, direction, and coordinate geometry
- 3.5 Mathematical reasoning in geometry

Data Analysis, Statistics, and Probability (32 objectives)

- 4.1 Data representation
- 4.2 Characteristics of data sets
- 4.3 Experiments and samples
- 4.4 Probability
- 4.5 Mathematical reasoning with data

Algebra (30 objectives)

- 5.1 Patterns, relations, and functions
- 5.2 Algebraic representations
- 5.3 Variables, expressions, and operations
- 5.4 Equations and inequalities
- 5.5 Mathematical reasoning in algebra

As indicated in the Introduction, the ACCUPLACER specifications used for the purpose of this analysis consist of the aggregated goals and objectives of three separate ACCUPLACER mathematics placement tests: Arithmetic, Elementary Algebra, and College Level Math. While each ACCUPLACER mathematics placement test would be taken separately by a student, it was determined that the aggregate of the three tests best represents the range of skills and knowledge a student would encounter on the ACCUPLACER companion forms selected for use in the NAEP–ACCUPLACER alignment study. The complete ACCUPLACER mathematics test specifications to be used in this study are, therefore, organized by test (represented at the standard level), followed by the respective goals and objectives for each assessment. ACCUPLACER standards and goals are listed below.[5] The number of objectives for each standard is listed, and the full text of the 87 objectives is included in Appendix A.

A. Arithmetic Test Content (28 objectives)
    - A.1 Whole Numbers and Fractions
    - A.2 Decimals and Percents
    - A.3 Applications

B. Elementary Algebra Test Content (19 objectives)
    - B.1 Integers and Rationals
    - B.2 Algebraic Expressions
    - B.3 Equations, Inequalities, and Word Problems

C. College Level Math Test Content (40 objectives)
    - C.1 Algebraic Operations
    - C.2 Solution of Equations and Inequalities
    - C.3 Coordinate Geometry
    - C.4 Applications and Other Algebra Topics
    - C.5 Functions
    - C.6 Trigonometry

The objectives under ACCUPLACER's three Arithmetic Test goals primarily assess basic computation skills, though some assess basic computation skills at a more complex, problem-solving level. The first of the three Elementary Algebra Test goals—Integers and Rationals—assesses some of the same skills found in the Arithmetic Test, while the second and third Elementary Algebra Test goals—Algebraic Expressions and Equations, Inequalities, and Word Problems—assess the basic skills of algebraic manipulation that may be acquired in the typical high school courses of Algebra I and Algebra II. The College Level Math Test goals primarily assess a more advanced set of algebra skills, typically acquired at the end of the high school curriculum and at the beginning of college-level study, as well as geometry and trigonometry.

[5] Source: Derived from data provided by the College Board.

The overall structures of the NAEP framework and the ACCUPLACER specifications differ substantially. The NAEP framework is organized around content topics—content that may commonly appear in assessment frameworks (e.g., state assessment frameworks, National Council of Teachers of Mathematics content strands). ACCUPLACER was developed to identify student placement; the combined specifications are organized developmentally. In addition, the NAEP framework is considerably more detailed and complex than the ACCUPLACER specifications, incorporating many more specific topics, skills, and content elements.

Nonetheless, areas of overlap between the ACCUPLACER specifications and the NAEP framework do exist. At the standard level, each ACCUPLACER standard has at least one objective that corresponds with a NAEP standard, goal, or objective, and vice versa. At the goal level, each ACCUPLACER goal has at least one objective that corresponds with a NAEP goal or objective; most ACCUPLACER goals, especially those for the Arithmetic Test and the Elementary Algebra Test, can be associated with NAEP goals found within either the Number Properties and Operations standard (Standard 1) or the Algebra standard (Standard 5). Six NAEP goals have no objectives that correspond with any ACCUPLACER goal or objective. They are:

1. Number Properties and Operations

1.5 Properties of number and operations

1.6 Mathematical reasoning using number

3. Geometry

3.5 Mathematical reasoning in geometry

4. Data Analysis, Statistics, and Probability

4.3 Experiments and samples

4.5 Mathematical reasoning with data

5. Algebra

5.5 Mathematical reasoning in algebra

At the objective level, certain ACCUPLACER objectives do not correspond with any NAEP objective, although they do cover content contained within a broader NAEP goal. The content of these ACCUPLACER objectives, however, is typically considered above the grade level represented by NAEP.

- ACCUPLACER C.4.b (“Complex numbers”), C.4.d (“Determinants”), and C.4.f (“Factorials”) do not correspond with any NAEP objectives, but do correspond with NAEP 1.1 (Number sense).
- ACCUPLACER C.5.a (“Functions of degree greater than 2”), C.5.e (“Composition of functions”), and C.6.h (“Inverse trigonometric functions”) do not correspond with any NAEP objectives, but do correspond with NAEP 5.1 (Patterns, relations, and functions).
- ACCUPLACER C.2.e (“Equations of degree greater than 2”) and C.6.e (“Trigonometric equations and inequalities”) do not correspond with any NAEP objectives, but do correspond with NAEP 5.4 (Equations and Inequalities).

Part of the difficulty in looking for full overlap between the NAEP framework and the ACCUPLACER specifications is the imbalance in overall size and scope between the two documents. Within a given mathematics strand, there may be only a few ACCUPLACER objectives available to correspond with a much larger set of NAEP goals. For example, NAEP 2 (Measurement) contains three goals and 18 objectives, whereas in ACCUPLACER, only five objectives, across two standards, address measurement (A.3.d, C.6.a, C.6.b, C.6.c, and C.6.f). No objectives within ACCUPLACER B (Elementary Algebra Test Content) reflect measurement content. A similar trend can be seen regarding NAEP 4 (Data Analysis, Statistics, and Probability) and NAEP 3 (Geometry).

Specificity of Content

As indicated previously, the NAEP framework and the ACCUPLACER specifications represent two divergent systems of mathematics organization. There is, however, some degree of overlap in the level of content specificity provided in the two documents, largely due to the descriptions provided at the ACCUPLACER goal level, which help explain the intent of corresponding objectives. These descriptions provide a level of *content* specificity that is comparable to that provided in the NAEP objectives. (There is, however, a discrepancy in the specificity of *complexity*, which will be discussed later in this section.) The presence of the goal-level descriptive text will inform alignment, facilitating the interpretation of ACCUPLACER objectives. For example, the first sentence in the description of ACCUPLACER B.2 ("Algebraic Expressions: Items in this category intend to measure the test-takers' ability to evaluate and manipulate algebraic expressions by applying field properties . . . and basic laws of exponents") helps to explain the intent of ACCUPLACER B.2.b ("Addition and subtraction of monomials and polynomials") and thus clarify its correspondence with the first part of NAEP 5.3.h ("Use basic properties of exponents and *logarithms to solve problems").[6] The presence of the descriptive text also serves to limit the scope of ACCUPLACER B.2.b, delineating, for example, that it will not encompass anything relating to the use of logarithms to solve problems (the second part of NAEP 5.3.h).

Although there is overlap in the level of specificity of content, the NAEP objectives tend to contain a higher level of specificity and thus are more narrowly focused than the ACCUPLACER objectives. For example, NAEP 1.3.c, "Perform arithmetic operations with expressions involving absolute value," provides more specificity regarding the nature of the task than the corresponding ACCUPLACER objective, B.1.c, "Absolute value." In other cases, NAEP objectives include content that goes beyond even a combination of ACCUPLACER objectives. For example, NAEP 1.1.d, "Represent, interpret, or compare expressions for real numbers, including expressions using exponents and logarithms," addresses the representation and interpretation of numbers and includes comparing expressions, while the two corresponding

[6] "Some of the grade 12 objectives are marked with an "*." This denotes objectives that describe mathematics content beyond that typically taught in a standard 3-year course of study (the equivalent of 1 year of geometry and 2 years of algebra). Therefore, these objectives will be selected less often than the others for inclusion on the assessments. Although all test items will be assigned a primary classification, some test items could potentially fall into more than one content area or under more than one objective" (National Assessment Governing Board, 2008, p. 7).

ACCUPLACER objectives—A.1.h and A.2.g ("Recognition of equivalent fractions and mixed numbers" and "Recognition of fraction, decimal, and percent equivalences," respectively)—address only the recognition of rational numbers. NAEP objectives may also be associated with ACCUPLACER objectives across multiple tests (i.e., Arithmetic, Elementary Algebra, and/or College Level Math). For example, NAEP 1.1.i, "Order or compare real numbers, including very large and very small real numbers," corresponds to ACCUPLACER A.2.h, "Ordering decimals, fractions, and percents; and rounding," and B.1.a, "Ordering."

As a further indication of the level of specificity described in the NAEP framework, the Governing Board has specified the intended distribution of cognitive complexity across NAEP items: 25% of NAEP items are to be at a low level of mathematical complexity, 50% are to be at a medium level of complexity, and 25% are to be at a high level of complexity (National Assessment Governing Board, 2008, p. 38). The wording of NAEP objectives reflects the intended level of complexity as well, clearly specifying the nature of the task to be performed. As examples of implied levels of cognitive complexity, one NAEP objective may require students to demonstrate reasoning about the mathematics described by an objective (a relatively sophisticated level of cognitive complexity), while another may require students to simply demonstrate proficiency in the skills related to an objective (a relatively low level of cognitive complexity). The NAEP levels of cognitive complexity are different from the Webb-developed depth of knowledge levels to be used in this study (Webb, 2009). However, the specificity of cognitive complexity intended by the distribution described above—and, more importantly, found in the text of the NAEP objectives themselves—will allow the NAEP framework to be coded for depth of knowledge.

On the other hand, the ACCUPLACER specifications do not provide this level of specificity with regard to complexity. At the most specific level of the ACCUPLACER specifications, objectives state a component of the corresponding goal but do not indicate the actual task to be completed in relation to the component. For example, ACCUPLACER A.1.f, "Mixed operations with whole numbers, fractions, and mixed numbers," does not indicate whether a student is to identify a mixed operation (a relatively low level of cognitive complexity), solve a double-step mathematics problem involving mixed operations (a somewhat higher level of cognitive complexity), or perform a series of steps involving mixed operations and requiring reasoning and planning (a relatively high level of cognitive complexity).

Due to this lack of specificity in the ACCUPLACER specifications, it was determined by the Governing Board and WestEd that there was insufficient information in the ACCUPLACER specifications for the objectives to be analyzed for their depth of knowledge level as part of the NAEP–ACCUPLACER mathematics alignment study. The ACCUPLACER test items can be coded for depth of knowledge, and the range of depth of knowledge can be reported, but the depth of knowledge consistency analysis cannot be conducted, given that the intended depth of knowledge level of the objectives cannot be reliably coded.

In summary, the NAEP objectives tend to describe the content more specifically than the ACCUPLACER objectives. NAEP objectives reflect a greater range of cognitive demand for the students, and may cross over standards to assess connections across concepts.

Student Performance

The NAEP framework and the ACCUPLACER specifications describe some similar content, with similar student performance expectations in those content areas they have in common. Both include objectives that require students to understand numbers and apply them through computation, estimation, and problem solving. Both also expect students to have a broad understanding of algebra.

The ACCUPLACER Arithmetic Test Content standard assesses a broad range of arithmetic skills necessary for high school mathematics, corresponding primarily with NAEP 1 (Number Properties and Operations). There is also one ACCUPLACER measurement objective that corresponds with three goals in NAEP 2 (Measurement). Three ACCUPLACER objectives correspond with two goals in NAEP 4 (Data Analysis and Probability).

The ACCUPLACER Elementary Algebra Test Content standard assesses a broad range of basic algebra skills necessary for the Algebra I and Algebra II courses in high school mathematics. The first goal addresses number; the second and third goals correspond with NAEP 5 (Algebra), although ACCUPLACER B.2.d ("Positive rational roots and exponents") corresponds with NAEP 1.3.a ("Find integral or simple fractional powers of real numbers").

The ACCUPLACER College Level Math Test Content standard assesses a broad range of the advanced algebra, functions, and trigonometry skills necessary for success in introductory college mathematics coursework, which correspond primarily with NAEP 5. A more superficial correspondence also exists with NAEP 2.3 ("Measurement in triangles"), and with NAEP 3 (Geometry).

Furthermore, the NAEP framework includes topics and skills not addressed by the ACCUPLACER specifications. These include NAEP objectives specifically addressing student knowledge of measurement; geometry; and data analysis, statistics, and probability.

Even in areas of overlapping content between the two tests, it appears that the two assessments differ in their expectations of student performance. While the differences in the degree of specificity and phrasing of objectives make it harder to identify differences in expectation, NAEP goals appear to require a higher degree of cognitive complexity than is required by comparable ACCUPLACER goals.

The differences in student performance expectations are also evident in the item types in each assessment. The NAEP framework calls for multiple-choice as well as short and extended constructed-response items, and the language of some of the NAEP objectives reflects the range of student performance possible in constructed-response item formats. For example, an item written to NAEP 3.5.c ("Analyze or explain a geometric argument by contradiction") would likely take the form of a constructed-response item. Within ACCUPLACER, however, all items are multiple-choice, thereby limiting the range of performance that students could demonstrate in relation to the skill(s) and/or knowledge represented in a given objective.

Number, Proportion, and Format of Items

The test designs for NAEP and ACCUPLACER differ greatly. There are 164 total mathematics items in the NAEP pool. NAEP is administered through a system of sampling; no single student completes all 164 items. Rather, each student completes two fixed "item sets" consisting of items

drawn from the larger NAEP pool (National Assessment Governing Board, 2007, p. 99). Twenty-five items in the 2009 NAEP mathematics item pool reference set leaders. Of these 25, 13 are constructed-response items and 12 are multiple-choice items. Between two and four items can refer to a single set leader. There are nine set leaders total. In contrast, ACCUPLACER is designed to be computer adaptive, with students completing items that are selected for them during the test based on their ongoing performance. ACCUPLACER is also available in a fixed "companion form" version. At the College Board's request, fixed forms were used in the alignment study. One 35-item form from each of the three placement tests provided was analyzed, for a total of 105 items. Each placement test consisted of 20 variable items unique to that form and 15 items common to a linked form (not common to the other placement tests).

As discussed previously, the NAEP framework and ACCUPLACER specifications differ in the format of items described. All (100%) of the 105 ACCUPLACER items are multiple-choice items. NAEP includes multiple-choice, short constructed-response, and extended constructed-response items. For NAEP, item distribution is expressed as the percentage of time students are expected to spend on each item type. For grade 12 mathematics, this distribution is divided equally between multiple-choice and constructed-response items (p. 97). In terms of the number of items in the total NAEP item pool to be included in the alignment study, the distribution is as follows: 66% (108) multiple choice; 27% (45) short constructed-response; and 7% (11) extended constructed-response.

The NAEP specifications do not correlate difficulty or complexity with item type. While the Governing Board admits a tendency toward increased cognitive demand on the part of constructed-response items, the NAEP specifications indicate that any item type may assess "mathematics of greater or less depth and sophistication" (p. 47).

Scoring Rubrics and Rules for Constructed-Response Items

Multiple-choice items in NAEP are scored as correct or incorrect. Constructed-response items in NAEP are scored according to the following rubrics:

- short constructed-response items are scored dichotomously: "correct or incorrect";
- short constructed-response items are scored on a three-point scale: "correct, partial, or incorrect";
- extended constructed-response items may have up to five scoring categories: "extensive, satisfactory, partial, minimal, or incorrect" (National Assessment Governing Board, 2008, pp. 80–84).

Every constructed-response item in NAEP has its own scoring rubric written specifically for that item.

As previously specified, ACCUPLACER does not include constructed-response items.

Grade Level Targeted by Items

According to the Governing Board, NAEP test questions should match the NAEP assessment framework and specifications, and accurately represent "the content domain to which inferences will be made" (National Assessment Governing Board, 2007, p. 116). "In broad terms, the

[NAEP] framework attempts to answer the question: What mathematics skills should be assessed in 2009 on NAEP at grades 4, 8, and 12?" (2008, p. 2). Consistent with this driving question, the grade 12 NAEP test specifications for mathematics have been revised in recent years to improve NAEP's ability to report on how well prepared grade 12 students are for postsecondary education and training.

The three ACCUPLACER mathematics tests are intended to be "used by college-level academic advisors and counselors to determine course selection" for incoming students. Thus the test is not targeted to a specific grade level but is designed to give college admissions and placement staff information about "the academic readiness of the students coming to you" (College Board, 2009a). The ACCUPLACER specifications do not provide information about the difficulty of items. However, some ACCUPLACER goals, mainly found in the set of goals for the College Level Math Test, go beyond what might be required of students completing the usual high school mathematics curriculum by grade 12.

Summary of Content Overlap and Implications for Item Alignment

This section presents a summary of the overlap in content between the NAEP mathematics framework and the ACCUPLACER mathematics specifications that was identified by the comparative analysis, indicating areas where the assessments appear to have greater or lesser degrees of potential alignment. The comparative analysis has raised a number of considerations regarding issues that may arise in the alignment study. These issues are described here, along with proposed decision rules to address them, if necessary.

Summary of Content Overlap

Table 1 shows a summary of the overlap between the ACCUPLACER and NAEP specifications at the goal level. The complete comparative analysis table for the alignment of ACCUPLACER to NAEP at the objective level, on which this summary is based, is included as Appendix A.

Table 1. Overlap of ACCUPLACER and NAEP Specifications at the Goal Level

ACCUPLACER	NAEP
A. Arithmetic Test Content A.1 Whole Numbers and Fractions	**1. Number Properties and Operations** 1.1 Number sense 1.2 Estimation 1.3 Number operations
A. Arithmetic Test Content A.2 Decimals and Percents	**1. Number Properties and Operations** 1.1 Number sense 1.3 Number operations 1.4 Ratios and proportional reasoning

ACCUPLACER	NAEP
A. Arithmetic Test Content A.3 Applications	**1. Number Properties and Operations** 1.3 Number operations 1.4 Ratios and proportional reasoning **2. Measurement** 2.1 Measuring physical attributes 2.2 Systems of measurement 2.3 Measurement in triangles **4. Data Analysis, Statistics, and Probability** 4.1 Data representation 4.2 Characteristics of data sets
B. Elementary Algebra Test Content B.1 Integers and Rationals	**1. Number Properties and Operations** 1.1 Number sense 1.3 Number operations
B. Elementary Algebra Test Content B.2 Algebraic Expressions	**1. Number Properties and Operations** 1.3 Number operations **5. Algebra** 5.3 Variables, expressions, and operations 5.4 Equations and inequalities
B. Elementary Algebra Test Content B.3 Equations, Inequalities, and Word Problems	**5. Algebra** 5.1 Patterns, relations, and functions 5.2 Algebraic representations 5.4 Equations and inequalities
C. College Level Math Test Content C.1 Algebraic Operations	**5. Algebra** 5.3 Variables, expressions, and operations 5.4 Equations and inequalities
C. College Level Math Test Content C.2 Solution of Equations and Inequalities	**4. Data Analysis, Statistics, and Probability** 4.2 Characteristics of data sets **5. Algebra** 5.2 Algebraic representations 5.4 Equations and inequalities
C. College Level Math Test Content C.3 Coordinate Geometry	**3. Geometry** 3.4 Position, direction, and coordinate geometry **5. Algebra** 5.1 Patterns, relations, and functions

ACCUPLACER	NAEP
C. College Level Math Test Content C.4 Applications and Other Algebra Topics	**3. Geometry** 3.1 Dimension and shape 3.2 Transformation of shapes and preservation of properties 3.3 Relationships between geometric figures **4. Data Analysis, Statistics, and Probability** 4.4 Probability **5. Algebra** 5.1 Patterns, relations, and functions 5.2 Algebraic representations
C. College Level Math Test Content C.5 Functions	**5. Algebra** 5.1 Patterns, relations, and functions 5.2 Algebraic representations 5.3 Variables, expressions, and operations 5.4 Equations and inequalities
C. College Level Math Test Content C.6 Trigonometry	**2. Measurement** 2.2 Systems of measurement 2.3 Measurement in triangles **5. Algebra** 5.1 Patterns, relations, and functions
Not aligned with any ACCUPLACER Topic	**1. Number Properties and Operations** 1.5 Properties of number and operations 1.6 Mathematical reasoning using number 3. **Geometry** 3.5 Mathematical reasoning in geometry **4. Data Analysis, Statistics, and Probability** 4.3 Experiments and samples 4.5 Mathematical reasoning with data **5. Algebra** 5.5 Mathematical reasoning in algebra

As shown in Table 1, all of the content represented by the three ACCUPLACER standards is represented by two or more NAEP goals. In some cases, the wording of an ACCUPLACER goal is very similar to the wording in the related NAEP goal. In other cases, it is inferred.

Although every ACCUPLACER goal has overlapping content with a NAEP standard, not every NAEP goal is represented in the ACCUPLACER specifications. Of the 24 NAEP goals, six are not represented by an ACCUPLACER goal. A more comprehensive comparison of the ACCUPLACER and NAEP objectives is found in Appendix A. A list of NAEP objectives with no corresponding ACCUPLACER objectives is located in Appendix B.

Implications for the Alignment Study and Related Potential Decision Rules

This section describes implications of the findings of this comparative analysis. Where potential decision rules have emerged, these are included below the related implication. The final decision rules used in the alignment study are included in Appendix C.

Considering items that contain operations involving only numbers and those involving variables

Objectives for the NAEP specifications include representations and operations both for numbers and variables. Items containing numbers should be primarily aligned to those objectives addressing numbers, and items containing variables should be primarily aligned to those objectives addressing algebra. As an example, ACCUPLACER C.1.f ("Powers, roots, radicals") is included within C.1 (Algebraic Operations). One would expect to see variables in items corresponding with this objective. Therefore, based on this decision rule, one would not determine ACCUPLACER C.1.f to correspond with NAEP 1.1.d ("Represent, interpret, or compare expressions for real numbers, including expressions using exponents and logarithms"), as this NAEP objective would likely address numerical expressions and not variables.

Proposed Decision Rule: The NAEP objectives within the Algebra standard will be interpreted as aligning primarily to items containing one or more variables and not items containing only numerical expressions. The NAEP objectives within the Number Operations and Properties standard will be interpreted as aligning primarily to numerical items; however, consideration is also to be given to items containing one or more variables.

Considering objectives that require the student to solve problems

The various NAEP objectives make reference to problem solving using wording that varies, but has similar connotations. Each of those objectives is interpreted to assess mathematics in situations involving either real-world problem solving or problem solving in a mathematical context. For example, it is determined that each of the following NAEP objectives implies problem solving tasks, even though the wording of each is somewhat different (italics added for emphasis):

- NAEP 1.1.g ("Represent, interpret, or compare expressions or *problem situations* involving absolute values")
- NAEP 1.2.c ("Verify solutions or determine the reasonableness of results in a *variety of situations*")
- NAEP 1.3.f ("*Solve application problems* involving numbers, including rational and common irrationals")
- NAEP 1.4.c ("Use proportions to *solve problems* (including rates of change)")
- NAEP 1.4.d ("*Solve multistep problems* involving percentages, including compound percentages")

Within the ACCUPLACER objectives, mention of problem solving tasks tends to appear in the description accompanying each goal. For example (italics added for emphasis):

- ACCUPLACER A.2 ("Decimals and Percents: These items measure the test-takers' computational fluency with mathematical operations . . . including *applications of properties* of numerical inequalities . . . ")

- ACCUPLACER A.3 ("Applications: In these items, test-takers are required to *solve word problems . . .*")
- ACCUPLACER B.2 ("Algebraic Expressions: Items in this category intend to measure the test-takers' ability to evaluate and manipulate algebraic expressions . . . including *applications of* squaring a binomial, factoring a difference of squares . . . ")
- ACCUPLACER Goal B.3 ("Equations, Inequalities, and Word Problems: . . . Some of these items can have an *abstract setting* and some others have a *concrete setting*. In the latter case, focus is on the test-takers' ability to translate verbal description of the *real-world situation,* or mathematical phrases, into algebraic expressions or equations, on ability to solve problems and interpreting of solutions *in context.*")

Proposed Decision Rule: The primary intent of objectives containing wording that refers to problem solving (e.g. solving problems, in contextual situations) is to assess mathematics in situations involving either real-world problem solving or problem solving in a mathematical context.

Considering objectives that have multiple parts

Some NAEP objectives contain multiple parts separated by the word "and." The intent of the objective is not necessarily interpreted to require that all parts assessed by each item align to that objective. For example, in NAEP 1.1.d ("Represent, interpret, or compare expressions for real numbers, including expressions using exponents and logarithms"), the "or" indicates that one would not be expected to do all three at once in the same item. The "and," however, could be interpreted as replaceable by an "or" (one would not use both exponents and logarithms). As another example, within NAEP 1.2.b ("Identify situations where estimation is appropriate, determine the needed degree of accuracy, and analyze* the effect of the estimation method on the accuracy of results"), the "and" preceding "analyze the effect" makes the interpretation of this objective somewhat ambiguous. As written, this objective implies that an item could require a student to identify, determine, and analyze; however, this intent of this objective is interpreted as inclusive of items that focus on any one—or combination—of the three tasks.

Proposed Decision Rule: If an item addresses only one part of the objective, panelists are asked to look for an alternative primary code. If an alternative code is not available, panelists are to note in the WAT that the item does not assess the entire objective.

Considering objectives that assess equations

Some NAEP objectives specifically address algebraic manipulations involving expressions, and others specifically address equations and inequalities. Since every equation and inequality contains two expressions, it is understood that those algebraic manipulations include expressions as well.

As an example, NAEP 5.3.c ("Perform basic operations, using appropriate tools, on algebraic expressions including polynomial and rational expressions") could be interpreted to include basic operations performed on equations as well as expressions; therefore, an ACCUPLACER item that involves equations but not expressions would still be determined to correspond with this NAEP objective.

Proposed Decision Rule: An objective that addresses expressions may be aligned with items containing equations if symbolic manipulation across the equal sign is not required to answer the question. An objective that addresses equations or inequalities may or may not also address expressions.

Considering duplicate objectives across the three ACCUPLACER tests

When coding an item to the ACCUPLACER test specifications, panelists will consider the intended audience for the various objectives. For example, various objectives are duplicated in Elementary Algebra and College Level Math. As an example, ACCUPLACER B.3.a ("Solving linear equations and inequalities") and C.2.a ("Linear equations and inequalities") are considered comparable except for the grade level.

Proposed Decision Rule: If the item assesses algebra at a basic level, the item should be aligned to an Elementary Algebra objective. If the item assesses algebra at an advanced level, the item should be aligned to a College Level Math objective.

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Appendix A: Objective-Level NAEP-to-ACCUPLACER Content Comparison

NAEP Mathematics	ACCUPLACER Mathematics	Similarities and Differences
1 Number properties and operations	Arithmetic Test: A.1 Whole Numbers and Fractions; A.2 Decimals and Percents; A.3 Applications; Elementary Algebra Test: B.1 Integers and Rationals	The NAEP standard is addressed by all three ACCUPLACER topics in the Arithmetic Test and one topic in the Elementary Algebra Test.
1.1 Number sense	C.4.b Complex numbers; C.4.d Determinants; C.4.f Factorials	The three ACCUPLACER objectives are not specifically addressed by any NAEP objectives, but are related to the NAEP goal of Number sense.
1.1.d Represent, interpret, or compare expressions for real numbers, including expressions using exponents and logarithms.	A.1.h Recognition of equivalent fractions and mixed numbers; A.2.g Recognition of fraction, decimal, and percent equivalences	The NAEP objective addresses real numbers in a broad sense. The ACCUPLACER objectives only address a subset of the real numbers at a basic level of understanding.
1.1.f Represent or interpret expressions involving very large or very small numbers in scientific notation.	A.2.c Multiplication of a decimal by a power of ten; A.2.f Division of a decimal by a power of ten	The NAEP objective only addresses the decimal number system concepts that involve scientific notation. The ACCUPLACER objectives address scientific notation at a prerequisite level.
1.1.g Represent, interpret, or compare expressions or problem situations involving absolute values.		No ACCUPLACER objectives describe the content of NAEP 1.1.g.
1.1.i Order or compare real numbers, including very large and very small real numbers.	A.2.h Ordering decimals, fractions, and percents; and rounding; B.1.a Ordering	The ACCUPLACER objectives only address ordering and comparing a subset of the real numbers at a basic level of understanding.
1.2 Estimation		
1.2.b Identify situations where estimation is appropriate, determine the needed degree of accuracy, and analyze* the effect of the estimation method on the accuracy of results.	A.1.a Computation and estimation of operations with whole numbers; A.1.i Estimation of sums, differences, products, and quotients that involve fractions or mixed numbers	The NAEP objective addresses the concepts of estimation in a broad sense involving complex understanding. The ACCUPLACER objectives only address estimation of whole numbers and fractions at a basic level of understanding.
1.2.c Verify solutions or determine the reasonableness of results in a variety of situations.	A.1.a Computation and estimation of operations with whole numbers; A.1.i Estimation of sums, differences, products, and quotients that involve fractions or mixed numbers	See NAEP 1.2.b.

NAEP Mathematics	ACCUPLACER Mathematics	Similarities and Differences
1.2.d Estimate square or cube roots of numbers less than 1,000 between two whole numbers.	A.1.g Squares and square roots (whole numbers, fractions); A.2.l Squares and square roots (decimals)	The NAEP objective addresses cube roots as well as square roots. The ACCUPLACER objectives are limited to squares and square roots.
1.3 Number operations		
1.3.a Find integral or simple fractional powers of real numbers.	B.2.d Positive rational roots and exponents; C.1.f Powers, roots, radicals	The NAEP objective addresses powers of real numbers and may include algebraic expressions. The ACCUPLACER objectives are listed under algebra topics and therefore are likely to only address algebraic expressions.
1.3.b Perform arithmetic operations with real numbers, including common irrational numbers.	A.1.a Computation and estimation of operations with whole numbers; A.1.b Addition and subtraction of fractions or mixed numbers; A.1.c Multiplication involving fractions or mixed numbers; A.1.d Division of a whole number by a fraction or mixed number; A.1.e Division of a fraction/mixed number by a whole number, fraction, or mixed number; A.1.f Mixed operations with whole numbers, fractions, and mixed numbers; A.2.a Addition and subtraction of decimals; A.2.b Multiplication of decimals by whole numbers other than powers of ten and by decimals; A.2.d Division of a decimal by a whole number other than a power of ten or by a decimal; A.2.e Division of a whole number by a decimal; A.3.c Distribution of a quantity into fractional parts; B.1.b Operations with signed numbers	The NAEP objective addresses computation of real numbers in a broad sense. The ACCUPLACER objectives only address a subset of the real numbers and do not include operations involving common irrational numbers.
1.3.c Perform arithmetic operations with expressions involving absolute value.	B.1.c Absolute value	The NAEP objective and the ACCUPLACER objective appear to address the same or closely related content.
1.3.d Describe the effect of multiplying and dividing by numbers including the effect of multiplying or dividing a real number by: Zero, or A number less than zero, or A number between zero and one, or One, or A number greater than one.	A.2.c Multiplication of a decimal by a power of ten; A.2.f Division of a decimal by a power of ten	The NAEP objective addresses the effects of the operations in a broad sense. The ACCUPLACER objectives only address operations involving powers of ten.

NAEP Mathematics	ACCUPLACER Mathematics	Similarities and Differences
1.3.f Solve application problems involving numbers, including rational and common irrationals.	A.1.a Computation and estimation of operations with whole numbers; A.1.b Addition and subtraction of fractions or mixed numbers; A.1.c Multiplication involving fractions or mixed numbers; A.1.d Division of a whole number by a fraction or mixed number; A.1.e Division of a fraction/mixed number by a whole number, fraction, or mixed number; A.1.f Mixed operations with whole numbers, fractions, and mixed numbers; A.2.a Addition and subtraction of decimals; A.2.b Multiplication of decimals by whole numbers other than powers of ten and by decimals; A.2.d Division of a decimal by a whole number other than a power of ten or by a decimal; A.2.e Division of a whole number by a decimal; A.3.c Distribution of a quantity into fractional parts; B.1.b Operations with signed numbers	The NAEP objective addresses applications of numbers in a broad sense. The various ACCUPLACER objectives may involve solving some real-world problems, but they imply using only rational numbers at a basic skill level.
1.4 Ratios and proportional reasoning		
1.4.c Use proportions to solve problems (including rates of change).	A.3.a Rate problems including ratio and proportion	The NAEP objective and the ACCUPLACER objective appear to address the same or closely related content.
1.4.d Solve multistep problems involving percentages, including compound percentages.	A.2.i Percent of a number; A.2.j Percent one number is of another; A.2.k A number when a percent of it is known; A.3.b Percent problems	The NAEP objective addresses solving percentage problems at a complex level. The ACCUPLACER objectives imply solving percent problems at a basic skill level.
1.5 Properties of number and operations		
1.5.c Solve problems using factors, multiples, or prime factorization.		No ACCUPLACER objectives describe the content of NAEP 1.5.c.
1.5.d Use divisibility or remainders in problem settings.		No ACCUPLACER objectives describe the content of NAEP 1.5.d.
1.5.e Apply basic properties of operations, including conventions about the order of operations.		No ACCUPLACER objectives describe the content of NAEP 1.5.e.

NAEP Mathematics	ACCUPLACER Mathematics	Similarities and Differences
1.5.f Recognize properties of the number system (whole numbers, integers, rational numbers, real numbers, and complex numbers) and how they are related to each other, and identify examples of each type of number.		No ACCUPLACER objectives describe the content of NAEP 1.5.f.
1.6 Mathematical reasoning using number		
1.6.a Give a mathematical argument to establish the validity of a simple numerical property or relationship.		No ACCUPLACER objectives describe the content of NAEP 1.6.a.
1.6.b * Analyze or interpret a proof by mathematical induction of a simple numerical relationship.		No ACCUPLACER objectives describe the content of NAEP 1.6.b.
2 Measurement	Arithmetic Test: A.3 Applications; College Level Math Test: C.6 Trigonometry	The NAEP standard is addressed by one ACCUPLACER topic in the Arithmetic Test and one topic in the College Level Math Test.
2.1 Measuring physical attributes		ACCUPLACER A.3.d is general in nature and is likely to describe several of the more specific NAEP objectives. The ACCUPLACER items may be characterized as Arithmetic Test content in a measurement context, whereas the NAEP objectives may tend to focus more on the actual content of the Measurement standard. In the instances in which a NAEP objective corresponds with A.3.d, that objective was judged to be typical of those for many large-scale exams and thus more likely to appear on the ACCUPLACER Arithmetic Test. In the instances in which a NAEP objective does not correspond with A.3.d, that objective was judged less likely to be assessed by a general set of Measurement problems.
2.1.b Determine the effect of proportions and scaling on length, area, and volume.		No ACCUPLACER objectives describe the content of NAEP 2.1.b (see NAEP 2.1).
2.1.c Estimate or compare perimeters or areas of two-dimensional geometric figures.	A.3.d Measurement problems	See NAEP 2.1.

NAEP Mathematics	ACCUPLACER Mathematics	Similarities and Differences
2.1.d Solve problems of angle measure, including those involving triangles or other polygons or parallel lines cut by a transversal.	A.3.d Measurement problems	See NAEP 2.1.
2.1.f Solve problems involving perimeter or area of plane figures such as polygons, circles, or composite figures.	A.3.d Measurement problems	See NAEP 2.1.
2.1.h Solve problems by determining, estimating, or comparing volumes or surface areas of three-dimensional figures.	A.3.d Measurement problems	See NAEP 2.1.
2.1.i Solve problems involving rates such as speed, density, population density, or flow rates.	A.3.d Measurement problems	See NAEP 2.1.
2.2 Systems of measurement		ACCUPLACER A.3.d is general in nature and is likely to describe several of the more specific NAEP objectives. The ACCUPLACER items may be characterized as Arithmetic Test content in a measurement context, whereas the NAEP objectives may tend to focus more on the actual content of the Measurement standard. In the instances in which a NAEP objective corresponds with A.3.d, that objective was judged to be typical of those for many large-scale exams and thus more likely to appear on the ACCUPLACER Arithmetic Test. In the instances in which a NAEP objective does not correspond with A.3.d, that objective was judged less likely to be assessed by a general set of Measurement problems.
2.2.a Recognize that geometric measurements (length, area, perimeter, and volume) depend on the choice of a unit, and apply such units in expressions, equations, and problem solutions.		No ACCUPLACER objectives describe the content of NAEP 2.2.a (see NAEP 2.2).
2.2.b Solve problems involving conversions within or between measurement systems, given the relationship between the units.	A.3.d Measurement problems	See NAEP 2.2.
2.2.d Understand that numerical values associated with measurements of physical quantities are approximate, are subject to variation, and must be assigned units of measurement.		No ACCUPLACER objectives describe the content of NAEP 2.2.d (see NAEP 2.2).

NAEP Mathematics	ACCUPLACER Mathematics	Similarities and Differences
2.2.e Determine appropriate accuracy of measurement in problem situations (e.g., the accuracy of measurement of the dimensions to obtain a specified accuracy of area) and find the measure to that degree of accuracy.	A.3.d Measurement problems	See NAEP 2.2.
2.2.f Construct or solve problems involving scale drawings.	A.3.d Measurement problems	See NAEP 2.2.
2.3 Measurement in triangles		ACCUPLACER A.3.d is general in nature and is likely to describe several of the more specific NAEP objectives. The ACCUPLACER items may be characterized as Arithmetic Test content in a measurement context, whereas the NAEP objectives may tend to focus more on the actual content of the Measurement standard. In the instances in which a NAEP objective corresponds with A.3.d, that objective was judged to be typical of those for many large-scale exams and thus more likely to appear on the ACCUPLACER Arithmetic Test.
2.3.a Solve problems involving indirect measurement.	A.3.d Measurement problems	See NAEP 2.3.
2.3.b Solve problems using the fact that trigonometric ratios (sine, cosine, and tangent) stay constant in similar triangles.		No ACCUPLACER objectives describe the content of NAEP 2.3.b (see NAEP 2.3).
2.3.c Use the definitions of sine, cosine, and tangent as ratios of sides in a right triangle to solve problems about length of sides and measure of angles.	C.6.a Fundamental definitions of trig functions e.g., sin, cos, tan, etc.	The NAEP objective addresses application of trigonometric definitions to solving right triangle problems. The ACCUPLACER objective may include some real-world problems, but it implies mostly basic recall or procedure items.
2.3.d Interpret and use the identity $\sin^2 \theta + \cos^2 \theta = 1$ for angles θ between 0° and 90°; recognize this identity as a special representation of the Pythagorean theorem.	C.6.b Right triangle trigonometry and circular functions	The NAEP objective and the ACCUPLACER objective appear to address the same or closely related content.
2.3.e * Determine the radian measure of an angle and explain how radian measurement is related to a circle of radius 1.		No ACCUPLACER objectives describe the content of NAEP 2.3.e (see NAEP 2.3).

NAEP Mathematics	ACCUPLACER Mathematics	Similarities and Differences
2.3.f * Use trigonometric formulas such as addition and double angle formulas.	C.6.f Trigonometric identities	The NAEP objective and the ACCUPLACER objective appear to address the same or closely related content.
2.3.g * Use the law of cosines and the law of sines to find unknown sides and angles of a triangle.	C.6.c Laws of sines and cosines	The NAEP objective and the ACCUPLACER objective appear to address the same or closely related content.
3 Geometry	College Level Math Test: C.3 Coordinate Geometry; C.4 Applications and Other Algebra Topics	The NAEP standard is addressed by two ACCUPLACER topics in the College Level Math Test.
3.1 Dimension and shape		ACCUPLACER C.4.g is general in nature and is likely to describe several of the more specific NAEP objectives. The ACCUPLACER items may be characterized as College Level Math Test content (Applications and Other Algebra Topics) in a geometric context, whereas the NAEP objectives may tend to focus more on the actual content of the Geometry standard. In the instances in which a NAEP objective corresponds with C.4.d, that objective was judged to be typical of those for many large-scale exams and thus more likely to appear on the ACCUPLACER College Level Math Test. In the instances in which a NAEP objective does not correspond with C.4.d, that objective was judged less likely to be assessed by a general set of items about polygons.
3.1.c Give precise mathematical descriptions or definitions of geometric shapes in the plane and in three-dimensional space.	C.4.g Polygons	See NAEP 3.1.
3.1.d Draw or sketch from a written description plane figures and planar images of three-dimensional figures.		No ACCUPLACER objectives describe the content of NAEP 3.1.d (see NAEP 3.1).
3.1.e Use two-dimensional representations of three-dimensional objects to visualize and solve problems.		No ACCUPLACER objectives describe the content of NAEP 3.1.e (see NAEP 3.1).
3.1.f Analyze properties of three-dimensional figures including spheres and hemispheres.		No ACCUPLACER objectives describe the content of NAEP 3.1.f (see NAEP 3.1).

NAEP Mathematics	ACCUPLACER Mathematics	Similarities and Differences
3.2 Transformation of shapes and preservation of properties		ACCUPLACER C.4.g is general in nature and is likely to describe several of the more specific NAEP objectives. The ACCUPLACER items may be characterized as College Level Math Test content (Applications and Other Algebra Topics) in a geometric context, whereas the NAEP objectives may tend to focus more on the actual content of the Geometry standard. In the instances in which a NAEP objective corresponds with C.4.d, that objective was judged to be typical of those for many large-scale exams and thus more likely to appear on the ACCUPLACER College Level Math Test. In the instances in which a NAEP objective does not correspond with C.4.d, that objective was judged less likely to be assessed by a general set of items about polygons.
3.2.a Recognize or identify types of symmetries (e.g., point, line, rotational, self-congruence) of two- and three-dimensional figures.		No ACCUPLACER objectives describe the content of NAEP 3.2.a (see NAEP 3.2).
3.2.b Give or recognize the precise mathematical relationship (e.g., congruence, similarity, orientation) between a figure and its image under a transformation.	C.4.g Polygons	See NAEP 3.2.
3.2.c Perform or describe the effect of a single transformation on two- and three-dimensional geometric shapes (reflections across lines of symmetry, rotations, translations, and dilations).		No ACCUPLACER objectives describe the content of NAEP 3.2.c (see NAEP 3.2).
3.2.d Identify transformations, combinations, or subdivisions of shapes that preserve the area of two-dimensional figures or the volume of three-dimensional figures.		No ACCUPLACER objectives describe the content of NAEP 3.2.d (see NAEP 3.2).
3.2.e Justify relationships of congruence and similarity and apply these relationships using scaling and proportional reasoning.	C.4.g Polygons	See NAEP 3.2.
3.2.g Perform or describe the effects of successive transformations.		No ACCUPLACER objectives describe the content of NAEP 3.2.g (see NAEP 3.2).

NAEP Mathematics	ACCUPLACER Mathematics	Similarities and Differences
3.3 Relationships between geometric figures		ACCUPLACER C.4.g is general in nature and is likely to describe several of the more specific NAEP objectives. The ACCUPLACER items may be characterized as College Level Math Test content (Applications and Other Algebra Topics) in a geometric context, whereas the NAEP objectives may tend to focus more on the actual content of the Geometry standard. In the instances in which a NAEP objective corresponds with C.4.d, that objective was judged to be typical of those for many large-scale exams and thus more likely to appear on the ACCUPLACER College Level Math Test. In the instances in which a NAEP objective does not correspond with C.4.d, that objective was judged less likely to be assessed by a general set of items about polygons.
3.3.b Apply geometric properties and relationships to solve problems in two and three dimensions.	C.4.g Polygons	See NAEP 3.3.
3.3.c Represent problem situations with geometric models to solve mathematical or real-world problems.	C.4.g Polygons	See NAEP 3.3.
3.3.d Use the Pythagorean theorem to solve problems in two- or three-dimensional situations.		No ACCUPLACER objectives describe the content of NAEP 3.3.d (see NAEP 3.3).
3.3.e Recall and interpret definitions and basic properties of congruent and similar triangles, circles, quadrilaterals, polygons, parallel, perpendicular and intersecting lines, and associated angle relationships.	C.4.g Polygons	See NAEP 3.3.
3.3.f Analyze properties or relationships of triangles, quadrilaterals, and other polygonal plane figures.		No ACCUPLACER objectives describe the content of NAEP 3.3.f (see NAEP 3.3).
3.3.g Analyze properties and relationships of parallel, perpendicular, or intersecting lines including the angle relationships that arise in these cases.		No ACCUPLACER objectives describe the content of NAEP 3.3.g (see NAEP 3.3).

NAEP Mathematics	ACCUPLACER Mathematics	Similarities and Differences
3.3.h Analyze properties of circles and the intersections of lines and circles (inscribed angles, central angles, tangents, secants, and chords).		No ACCUPLACER objectives describe the content of NAEP 3.3.h (see NAEP 3.3).
3.4 Position, direction, and coordinate geometry		
3.4.a Solve problems involving the coordinate plane such as the distance between two points, the midpoint of a segment, or slopes of perpendicular or parallel lines.	C.3.a The coordinate plane; C.3.b Straight lines	The NAEP objective and ACCUPLACER C.3.a appear to address the same content. ACCUPLACER C.3.b may address concepts of the coordinate plane as they relate to parallel and perpendicular lines.
3.4.b Describe the intersections of lines in the plane and in space, intersections of a line and a plane, or of two planes in space.		No ACCUPLACER objectives describe the content of NAEP 3.4.b.
3.4.c Describe or identify conic sections and other cross-sections of solids.	C.3.c Conics	The NAEP objective addresses conic sections and cross-sections in a geometric context of shapes. The ACCUPLACER objective likely only addresses cross-sections in the coordinate plane as they relate to algebraic representation. However, there may be some overlap in basic concepts between the two objectives.
3.4.d Represent two-dimensional figures algebraically using coordinates and/or equations.	C.3.a The coordinate plane	The NAEP objective addresses geometric figures on the coordinate plane. The ACCUPLACER objective addresses the coordinate plane in a general sense and likely includes items that are not two-dimensional figures.
3.4.e * Use vectors to represent velocity and direction; multiply a vector by a scalar and add vectors both algebraically and graphically.		No ACCUPLACER objectives describe the content of NAEP 3.4.e.
3.4.f Find an equation of a circle given its center and radius and, given an equation of a circle, find its center and radius.	C.3.d Locus of points	The NAEP objective addresses a subset of the ACCUPLACER objective. Combined with NAEP 3.4.g, the two NAEP objectives appear to address the same or very similar content as the ACCUPLACER objective.

NAEP Mathematics	ACCUPLACER Mathematics	Similarities and Differences
3.4.g * Graph ellipses and hyperbolas whose axes are parallel to the coordinate axes and demonstrate understanding of the relationship between their standard algebraic form and their graphical characteristics.	C.3.d Locus of points	See NAEP 3.4.f.
3.4.h * Represent situations and solve problems involving polar coordinates.		No ACCUPLACER objectives describe the content of NAEP 3.4.h.
3.5 Mathematical reasoning in geometry		
3.5.a Make, test, and validate geometric conjectures using a variety of methods including deductive reasoning and counterexamples.		No ACCUPLACER objectives describe the content of NAEP 3.5.a.
3.5.b Determine the role of hypotheses, logical implications, and conclusion in proofs of geometric theorems.		No ACCUPLACER objectives describe the content of NAEP 3.5.b.
3.5.c Analyze or explain a geometric argument by contradiction.		No ACCUPLACER objectives describe the content of NAEP 3.5.c.
3.5.d Analyze or explain a geometric proof of the Pythagorean theorem.		No ACCUPLACER objectives describe the content of NAEP 3.5.d.
3.5.e Prove basic theorems about congruent and similar triangles and circles.		No ACCUPLACER objectives describe the content of NAEP 3.5.e.
4 Data analysis, statistics, and probability	Arithmetic Test: A.3 Applications; College Level Math Test: C.2 Solution of Equations and Inequalities; C.4 Applications and Other Algebra Topics	The NAEP standard is addressed by one ACCUPLACER topic in the Arithmetic Test and one topic in the College Level Math Test.

NAEP Mathematics	ACCUPLACER Mathematics	Similarities and Differences
4.1 Data representation		ACCUPLACER A.3.f is general in nature and is likely to describe one or more specific NAEP objectives. The ACCUPLACER items may be characterized as Arithmetic Test content in a data analysis context, whereas the NAEP objectives may tend to focus more on the actual content of the Data Analysis, Statistics, and Probability standard. In the instances in which a NAEP objective corresponds with A.3.f, that objective was judged to be typical of those for many large-scale exams and thus more likely to appear on the ACCUPLACER Arithmetic Test. In the instances in which a NAEP objective does not correspond with A.3.f, that objective was judged less likely to be assessed by a general set of items about graphs and tables.
4.1.a Read or interpret graphical or tabular representations of data.		No ACCUPLACER objectives describe the content of NAEP 4.1.a (see NAEP 4.1).
4.1.b For a given set of data, complete a graph and solve a problem using the data in the graph (histograms, scatterplots, and line graphs).	A.3.f Problems related to graphs and tables	See NAEP 4.1.
4.1.c Solve problems involving univariate or bivariate data.	A.3.f Problems related to graphs and tables	See NAEP 4.1.
4.1.d Given a graphical or tabular representation of a set of data, determine whether information is represented effectively and appropriately.		No ACCUPLACER objectives describe the content of NAEP 4.1.d (see NAEP 4.1).
4.1.e Compare and contrast different graphical representations of univariate and bivariate data.		No ACCUPLACER objectives describe the content of NAEP 4.1.e (see NAEP 4.1).
4.1.f Organize and display data in a spreadsheet in order to recognize patterns and solve problems.	A.3.f Problems related to graphs and tables	See NAEP 4.1.

NAEP Mathematics	ACCUPLACER Mathematics	Similarities and Differences
4.2 Characteristics of data sets		The objectives for ACCUPLACER A.3 and C.2 are general in nature and are likely to describe one or more specific NAEP objectives. The ACCUPLACER items may be characterized as Arithmetic Test and College Level Math Test content in a data analysis context, whereas the NAEP objectives may tend to focus more on the actual content of the Data Analysis, Statistics, and Probability standard. In the instances in which a NAEP objective corresponds with an ACCUPLACER objective, that objective was judged to be typical of those for many large-scale exams and thus more likely to appear on the ACCUPLACER Arithmetic Test or College Level Math Test. In the instances in which a NAEP objective does not correspond with an ACCUPLACER objective, that objective was judged less likely to be assessed by a general set of items about characteristics of data sets.
4.2.a Calculate, interpret, or use summary statistics for distributions of data including measures of typical value (mean, median), position (quartiles, percentiles), and spread (range, interquartile range, variance, and standard deviation).	A.3.e Average	See NAEP 4.2.
4.2.b Recognize how linear transformations of one-variable data affect mean, median, mode, range, interquartile range, and standard deviation.		No ACCUPLACER objectives describe the content of NAEP 4.2.b (see NAEP 4.2).
4.2.c Determine the effect of outliers on mean, median, mode, range, interquartile range, or standard deviation.	A.3.g Other, less routine problems	See NAEP 4.2.
4.2.d Compare data sets using summary statistics (mean, median, mode, range, interquartile range, or standard deviation) describing the same characteristic for two different populations or subsets of the same population.	A.3.e Average	See NAEP 4.2.

NAEP Mathematics	ACCUPLACER Mathematics	Similarities and Differences
4.2.e Approximate a trend line if a linear pattern is apparent in a scatterplot or use a graphing calculator to determine a least-squares regression line and use the line or equation to make predictions.	C.2.a Linear equations and inequalities	See NAEP 4.2.
4.2.f Recognize that the correlation coefficient is a number from -1 to +1 that measures the strength of the linear relationship between two variables; visually estimate the correlation coefficient (e.g., positive or negative, closer to 0, .5, or 1.0) of a scatterplot.		No ACCUPLACER objectives describe the content of NAEP 4.2.f (see NAEP 4.2).
4.2.g Know and interpret the key characteristics of a normal distribution such as shape, center (mean), and spread (standard deviation).		No ACCUPLACER objectives describe the content of NAEP 4.2.g (see NAEP 4.2).
4.3 Experiments and samples		
4.3.a Identify possible sources of bias in sample surveys and describe how such bias can be controlled and reduced.		No ACCUPLACER objectives describe the content of NAEP 4.3.a.
4.3.b Recognize and describe a method to select a simple random sample.		No ACCUPLACER objectives describe the content of NAEP 4.3.b.
4.3.c * Draw inferences from samples, such as estimates of proportions in a population, estimates of population means, or decisions about differences in means for two “treatments.”		No ACCUPLACER objectives describe the content of NAEP 4.3.c.
4.3.d Identify or evaluate the characteristics of a good survey or of a well-designed experiment.		No ACCUPLACER objectives describe the content of NAEP 4.3.d.
4.3.e * Recognize the differences in design and in conclusions between randomized experiments and observational studies.		No ACCUPLACER objectives describe the content of NAEP 4.3.e.
4.4 Probability		

NAEP Mathematics	ACCUPLACER Mathematics	Similarities and Differences
4.4.a Recognize whether two events are independent or dependent.		No ACCUPLACER objectives describe the content of NAEP 4.4.a.
4.4.b Determine the theoretical probability of simple and compound events in familiar or unfamiliar contexts.		No ACCUPLACER objectives describe the content of NAEP 4.4.b.
4.4.c Given the results of an experiment or simulation, estimate the probability of simple or compound events in familiar or unfamiliar contexts.		No ACCUPLACER objectives describe the content of NAEP 4.4.c.
4.4.d Use theoretical probability to evaluate or predict experimental outcomes.		No ACCUPLACER objectives describe the content of NAEP 4.4.d.
4.4.e Determine the number of ways an event can occur using tree diagrams, formulas for combinations and permutations, or other counting techniques.	C.4.e Permutations and combinations	The NAEP objective addresses outcomes of events in a broad sense. The ACCUPLACER objective only addresses a subset of the concepts of outcomes, but at a high level.
4.4.h Determine the probability of independent and dependent events.		No ACCUPLACER objectives describe the content of NAEP 4.4.h.
4.4.i Determine conditional probability using two-way tables.		No ACCUPLACER objectives describe the content of NAEP 4.4.i.
4.4.j Interpret and apply probability concepts to practical situations.		No ACCUPLACER objectives describe the content of NAEP 4.4.j.
4.4.k *Use the binomial theorem to solve problems.		No ACCUPLACER objectives describe the content of NAEP 4.4.k.
4.5 Mathematical reasoning with data		
4.5.a Identify misleading uses of data in real-world settings and critique different ways of presenting and using information.		No ACCUPLACER objectives describe the content of NAEP 4.5.a.
4.5.b Distinguish relevant from irrelevant information, identify missing information, and either find what is needed or make appropriate approximations.		No ACCUPLACER objectives describe the content of NAEP 4.5.b.
4.5.c *Recognize, use, and distinguish between the processes of mathematical (deterministic) and statistical modeling.		No ACCUPLACER objectives describe the content of NAEP 4.5.c.

NAEP Mathematics	ACCUPLACER Mathematics	Similarities and Differences
4.5.d Recognize when arguments based on data confuse correlation with causation.		No ACCUPLACER objectives describe the content of NAEP 4.5.d.
4.5.e * Recognize and explain the potential errors caused by extrapolating from data.		No ACCUPLACER objectives describe the content of NAEP 4.5.e.
5 Algebra	Elementary Algebra Test: B.2 Algebraic Expressions; B.3 Equations, Inequalities, and Word Problems; College Level Math Test: C.1 Algebraic Operations; C.2 Solution of Equations and Inequalities; C.3 Coordinate Geometry; C.4 Applications and Other Algebra Topics; C.5 Functions; C.6 Trigonometry	The NAEP standard is addressed by two of the three ACCUPLACER topics in the Elementary Algebra Test and all six topics in the College Level Math Test.
5.1 Patterns, relations, and functions	C.5.a Functions of degree greater than 2; C.5.e Composition of functions; C.6.h Inverse trigonometric functions	The three ACCUPLACER objectives are not specifically addressed by any NAEP objectives, but are related to the NAEP goal of Patterns, relations, and functions.
5.1.a Recognize, describe, or extend numerical patterns, including arithmetic and geometric progressions.	C.4.c Series and Sequences	The NAEP objective addresses patterning in a broad sense. The ACCUPLACER objective only addresses a subset of the concepts of patterns, but at a high level.
5.1.b Express linear and exponential functions in recursive and explicit form given a table, verbal description, or some terms of a sequence.		No ACCUPLACER objectives describe the content of NAEP 5.1.b.
5.1.e Identify or analyze distinguishing properties of linear, quadratic, rational, exponential, or *trigonometric functions from tables, graphs, or equations.	B.3.f Graphing; C.3.e Graphs of algebraic functions; C.6.d Graphs of trigonometric functions	The NAEP objective addresses the properties of algebraic functions in a broad sense involving various representations. The ACCUPLACER objectives address a subset of function concepts: the graphs of functions.
5.1.g Determine whether a relation, given in verbal, symbolic, tabular, or graphical form, is a function.		No ACCUPLACER objectives describe the content of NAEP 5.1.g.

NAEP Mathematics	ACCUPLACER Mathematics	Similarities and Differences
5.1.h Recognize and analyze the general forms of linear, quadratic, rational, exponential, or *trigonometric functions.	B.3.f Graphing; C.3.e Graphs of algebraic functions; C.6.d Graphs of trigonometric functions; C.6.g Trigonometric functions of two angles	The NAEP objective addresses the general forms of algebraic functions in a broad sense. The ACCUPLACER objectives address a subset of function concepts: the graphs of functions.
5.1.i Determine the domain and range of functions given in various forms and contexts.	C.5.d Domain and range	The NAEP objective and the ACCUPLACER objective appear to address the same or closely related content.
5.1.j * Given a function, determine its inverse if it exists and explain the contextual meaning of the inverse for a given situation.	C.5.f Inverse functions	The NAEP objective and the ACCUPLACER objective appear to address the same or closely related content.
5.2 Algebraic representations		
5.2.a Create and translate between different representations of algebraic expressions, equations, and inequalities (e.g., linear, quadratic, exponential, or *trigonometric) using symbols, graphs, tables, diagrams, or written descriptions.	C.4.a Translation	The NAEP objective and the ACCUPLACER objective appear to address the same or closely related content.
5.2.b Analyze or interpret relationships expressed in symbols, graphs, tables, diagrams (including Venn diagrams), or written descriptions and evaluate the relative advantages or disadvantages of different representations to answer specific questions.		No ACCUPLACER objectives describe the content of NAEP 5.2.b.
5.2.d Perform or interpret transformations on the graphs of linear, quadratic, exponential, and *trigonometric functions.	B.3.f Graphing; C.4.a Translation; C.5.c Graphical properties, exponential and logarithmic functions; C.5.h Periodicity, amplitude, and other properties	The NAEP objective addresses transformations of algebraic functions in a broad sense, but may only involve a subset of the combined list of ACCUPLACER objectives.
5.2.e Make inferences or predictions using an algebraic model of a situation.	C.2.a Linear equations and inequalities	The NAEP objective addresses the application of algebraic functions in a real-world context. The ACCUPLACER objective addresses only linear algebraic modeling.

NAEP Mathematics	ACCUPLACER Mathematics	Similarities and Differences
5.2.f Given a real-world situation, determine if a linear, quadratic, rational, exponential, logarithmic, or *trigonometric function fits the situation.	C.2.a Linear equations and inequalities	See NAEP 5.2.e.
5.2.g Solve problems involving exponential growth and decay.	C.5.b Exponents and logarithms	The NAEP objective addresses the application of exponential functions in a real-world context. The ACCUPLACER objective likely addresses the concepts of exponents and logarithms at a basic skill level.
5.2.h * Analyze properties of exponential, logarithmic, and rational functions.	C.5.b Exponents and logarithms; C.5.c Graphical properties, exponential and logarithmic functions	The NAEP objective addresses the properties of the three functions in a broad sense and likely at a high level of complexity. The ACCUPLACER objectives likely address the concepts of exponents and logarithms (and not rational functions) at a more basic skill level.
5.3 Variables, expressions, and operations		
5.3.b Write algebraic expressions, equations, or inequalities to represent a situation.	B.3.d Translating written phrases or sentences into algebraic expressions or equations	The NAEP objective and the ACCUPLACER objective appear to address the same or closely related content.
5.3.c Perform basic operations, using appropriate tools, on algebraic expressions including polynomial and rational expressions.	B.2.b Addition and subtraction of monomials and polynomials; B.2.c Multiplication of monomials and polynomials; B.2.e Squaring a binomial; B.2.f Factoring difference of squares; B.2.g Factoring $ax^2 + bx + c$ over the integers; B.2.h Factoring polynomials that are not quadratics; B.2.i Operations with algebraic fractions involving addition, subtraction, multiplication, and division; B.2.j Division of monomials and polynomials including simplification of algebraic fractions; C.1.a Addition of algebraic fractions; C.1.c Operations with polynomials; C.1.d Multiplication, division, and simplification of algebraic fractions; C.1.e Operations with exponents; C.1.g Factoring quadratic expressions	The NAEP objective and the ACCUPLACER objectives appear to address the same or closely related content.

NAEP Mathematics	ACCUPLACER Mathematics	Similarities and Differences
5.3.d Write equivalent forms of algebraic expressions, equations, or inequalities to represent and explain mathematical relationships.	B.2.e Squaring a binomial; B.2.f Factoring difference of squares; B.2.g Factoring $ax^2 + bx + c$ over the integers; B.2.h Factoring polynomials that are not quadratics	The NAEP objective addresses the concepts of equivalent forms of representation in a broad sense. The ACCUPLACER objectives only address the skills related to factoring.
5.3.e Evaluate algebraic expressions including polynomials and rational expressions.	B.2.a Evaluating formulas and other algebraic expressions	The NAEP objective and the ACCUPLACER objective appear to address the same or closely related content.
5.3.f Use function notation to evaluate a function at a specified point in its domain and combine functions by addition, subtraction, multiplication, division, and composition.	C.5.g Computations with simple functions	The NAEP objective and the ACCUPLACER objective appear to address the same or closely related content.
5.3.g * Determine the sum of finite and infinite arithmetic and geometric series.		No ACCUPLACER objectives describe the content of NAEP 5.3.g.
5.3.h Use basic properties of exponents and *logarithms to solve problems.	B.2.d Positive rational roots and exponents; C.1.e Operations with exponents	The NAEP objective addresses the properties of exponents and logarithms in a broad sense. The ACCUPLACER objectives only address the skills related to exponents.
5.4 Equations and inequalities	C.2.e Equations of degree greater than 2; C.6.e Trigonometric equations and inequalities	The two ACCUPLACER objectives are not specifically addressed by any NAEP objectives, but are related to the NAEP goal of Equations and inequalities.
5.4.a Solve linear, rational, or quadratic equations or inequalities, including those involving absolute value.	B.3.a Solving linear equations and inequalities; B.3.c Quadratic equations solution by factoring; C.1.b Addition and subtraction of expressions involving absolute value; C.2.a Linear equations and inequalities; C.2.b Quadratic equations	The NAEP objective addresses solving equations and inequalities in a broad sense. The ACCUPLACER objectives address only part of the concepts of solving absolute value equations and inequalities, and they do not address solving rational equations or inequalities.
5.4.c Analyze situations, develop mathematical models, or solve problems using linear, quadratic, exponential, or logarithmic equations or inequalities symbolically or graphically.	B.3.d Translating written phrases or sentences into algebraic expressions or equations; B.3.e Solving verbal problems in an algebraic context including geometric reasoning; B.3.f Graphing; C.2.d Exponential equations; C.4.a Translation	The NAEP objective addresses algebraic modeling and solving real-world problems in a broad sense using various types of equations and inequalities. The ACCUPLACER objectives appear to address the same or very similar content as the NAEP objective, but do not specifically address logarithms.
5.4.d Solve (symbolically or graphically) a system of equations or inequalities and recognize the relationship between the analytical solution and graphical solution.	B.3.b Systems of linear equations; C.2.c Systems of equations and inequalities	The NAEP objective and the ACCUPLACER objective appear to address the same or closely related content.

NAEP Mathematics	ACCUPLACER Mathematics	Similarities and Differences
5.4.e Solve problems involving special formulas such as: $A = P(I + r)^t$, $A = Pe^{rt}$.		No ACCUPLACER objectives describe the content of NAEP 5.4.e.
5.4.f Solve an equation or formula involving several variables for one variable in terms of the others.		No ACCUPLACER objectives describe the content of NAEP 5.4.f.
5.4.g Solve quadratic equations with complex roots.	C.2.b Quadratic equations	The NAEP objective and the ACCUPLACER objective appear to address the same or closely related content, although the NAEP objective is likely a subset of the concepts covered by the ACCUPLACER objective.
5.5 Mathematical reasoning in algebra		
5.5.a Use algebraic properties to develop a valid mathematical argument.		No ACCUPLACER objectives describe the content of NAEP 5.5.a.
5.5.b Determine the role of hypotheses, logical implications, and conclusions in algebraic argument.		No ACCUPLACER objectives describe the content of NAEP 5.5.b.
5.5.c Explain the use of relational conjunctions (and, or) in algebraic arguments.		No ACCUPLACER objectives describe the content of NAEP 5.5.c.

Appendix B: NAEP Objectives Not Addressed in the ACCUPLACER Specifications

The 64 NAEP objectives not addressed in the ACCUPLACER specifications are listed below.

1.1.g Represent, interpret, or compare expressions or problem situations involving absolute values.

1.5.c Solve problems using factors, multiples, or prime factorization.

1.5.d Use divisibility or remainders in problem settings.

1.5.e Apply basic properties of operations, including conventions about the order of operations.

1.5.f Recognize properties of the number system (whole numbers, integers, rational numbers, real numbers, and complex numbers) and how they are related to each other, and identify examples of each type of number.

1.6.a Give a mathematical argument to establish the validity of a simple numerical property or relationship.

1.6.b *Analyze or interpret a proof by mathematical induction of a simple numerical relationship.

2.1.b Determine the effect of proportions and scaling on length, area, and volume.

2.2.a Recognize that geometric measurements (length, area, perimeter, and volume) depend on the choice of a unit, and apply such units in expressions, equations, and problem solutions.

2.2.d Understand that numerical values associated with measurements of physical quantities are approximate, are subject to variation, and must be assigned units of measurement.

2.3.b Solve problems using the fact that trigonometric ratios (sine, cosine, and tangent) stay constant in similar triangles.

2.3.e *Determine the radian measure of an angle and explain how radian measurement is related to a circle of radius 1.

3.1.d Draw or sketch from a written description plane figures and planar images of three-dimensional figures.

3.1.e Use two-dimensional representations of three-dimensional objects to visualize and solve problems.

3.1.f Analyze properties of three-dimensional figures including spheres and hemispheres.

3.2.a Recognize or identify types of symmetries (e.g., point, line, rotational, self-congruence) of two- and three-dimensional figures.

3.2.c Perform or describe the effect of a single transformation on two- and three-dimensional geometric shapes (reflections across lines of symmetry, rotations, translations, and dilations).

3.2.d Identify transformations, combinations, or subdivisions of shapes that preserve the area of two-dimensional figures or the volume of three-dimensional figures.

3.2.g Perform or describe the effects of successive transformations.

3.3.d Use the Pythagorean theorem to solve problems in two- or three-dimensional situations.

3.3.f Analyze properties or relationships of triangles, quadrilaterals, and other polygonal plane figures.

3.3.g Analyze properties and relationships of parallel, perpendicular, or intersecting lines including the angle relationships that arise in these cases.

3.3.h Analyze properties of circles and the intersections of lines and circles (inscribed angles, central angles, tangents, secants, and chords).

3.4.b Describe the intersections of lines in the plane and in space, intersections of a line and a plane, or of two planes in space.

3.4.e *Use vectors to represent velocity and direction; multiply a vector by a scalar and add vectors both algebraically and graphically.

3.4.h *Represent situations and solve problems involving polar coordinates.

3.5.a Make, test, and validate geometric conjectures using a variety of methods including deductive reasoning and counterexamples.

3.5.b Determine the role of hypotheses, logical implications, and conclusion in proofs of geometric theorems.

3.5.c Analyze or explain a geometric argument by contradiction.

3.5.d Analyze or explain a geometric proof of the Pythagorean theorem.

3.5.e Prove basic theorems about congruent and similar triangles and circles.

4.1.a Read or interpret graphical or tabular representations of data.

4.1.d Given a graphical or tabular representation of a set of data, determine whether information is represented effectively and appropriately.

4.1.e Compare and contrast different graphical representations of univariate and bivariate data.

4.2.b Recognize how linear transformations of one-variable data affect mean, median, mode, range, interquartile range, and standard deviation.

4.2.f Recognize that the correlation coefficient is a number from -1 to +1 that measures the strength of the linear relationship between two variables; visually estimate the correlation coefficient (e.g., positive or negative, closer to 0, .5, or 1.0) of a scatterplot.

4.2.g Know and interpret the key characteristics of a normal distribution such as shape, center (mean), and spread (standard deviation).

4.3.a Identify possible sources of bias in sample surveys and describe how such bias can be controlled and reduced.

4.3.b Recognize and describe a method to select a simple random sample.

4.3.c *Draw inferences from samples, such as estimates of proportions in a population, estimates of population means, or decisions about differences in means for two "treatments."

4.3.d Identify or evaluate the characteristics of a good survey or of a well-designed experiment.

4.3.e *Recognize the differences in design and in conclusions between randomized experiments and observational studies.

4.4.a Recognize whether two events are independent or dependent.

4.4.b Determine the theoretical probability of simple and compound events in familiar or unfamiliar contexts.

4.4.c Given the results of an experiment or simulation, estimate the probability of simple or compound events in familiar or unfamiliar contexts.

4.4.d Use theoretical probability to evaluate or predict experimental outcomes.

4.4.h Determine the probability of independent and dependent events.

4.4.i Determine conditional probability using two-way tables.

4.4.j Interpret and apply probability concepts to practical situations.

4.4.k *Use the binomial theorem to solve problems.

4.5.a Identify misleading uses of data in real-world settings and critique different ways of presenting and using information.

4.5.b Distinguish relevant from irrelevant information, identify missing information, and either find what is needed or make appropriate approximations.

4.5.c *Recognize, use, and distinguish between the processes of mathematical (deterministic) and statistical modeling.

4.5.d Recognize when arguments based on data confuse correlation with causation.

4.5.e *Recognize and explain the potential errors caused by extrapolating from data.

5.1.b Express linear and exponential functions in recursive and explicit form given a table, verbal description, or some terms of a sequence.

5.1.g Determine whether a relation, given in verbal, symbolic, tabular, or graphical form, is a function.

5.2.b Analyze or interpret relationships expressed in symbols, graphs, tables, diagrams (including Venn diagrams), or written descriptions and evaluate the relative advantages or disadvantages of different representations to answer specific questions.

5.4.e Solve problems involving special formulas such as: $A = P(I + r)^t$, $A = Pe^{rt}$.

5.3.g *Determine the sum of finite and infinite arithmetic and geometric series.

5.4.f Solve an equation or formula involving several variables for one variable in terms of the others.

5.5.a Use algebraic properties to develop a valid mathematical argument.

5.5.b Determine the role of hypotheses, logical implications, and conclusions in algebraic argument.

5.5.c Explain the use of relational conjunctions (and, or) in algebraic arguments.

Appendix C: Decision Rules Applied in Operational Study

NAEP MATHEMATICS FRAMEWORK FOR ALIGNMENT: DECISION RULES

1) The objectives within the Algebra standard will be interpreted as aligning primarily to items containing one or more variables and not items containing only numerical expressions.
2) The objectives within the Number Operations and Properties standard will be interpreted as aligning primarily to numerical items; however, consideration is also to be given to items containing one or more variables.
3) The primary intent of objectives containing wording similar to the following is to assess mathematics in situations involving either real-world problem solving or problem solving in a mathematical context.
    - 1.1.g Represent, interpret, or compare expressions or problem situations involving absolute values.
    - 1.3.f Solve application problems involving numbers, including rational and common irrationals.
    - 1.4.c Use proportions to solve problems (including rates of change).
4) Some objectives contain multiple parts separated by the word "and" (see 1.5.f below). The intent of the objective may or may not be to assess all parts. If an item addresses only one part of the objective, panelists are asked to look for an alternative primary code. If an alternative is not available, panelists are to note in the WAT that the item does not assess the entire objective.
    - 1.5.f Recognize properties of the number system (whole numbers, integers, rational numbers, real numbers, and complex numbers) and how they are related to each other, and identify examples of each type of number.
5) An objective that addresses expressions may also be aligned with an item containing an equation if symbolic manipulation across the equal sign is not required to answer the question.

ACCUPLACER MATHEMATICS FRAMEWORK FOR ALIGNMENT: DECISION RULES

1) When coding an item, panelists will consider the intended audience for objectives. For example, a likely place for a decision of this sort is among the algebra objectives shared by both the Elementary Algebra Test and the College Level Math Test. If the item assesses algebra at a basic level, the item should be aligned to an Elementary Algebra objective. If the item assesses algebra at an advanced level, the item should be aligned to a College Level Math objective.

2) As with the NAEP objectives, the primary intent of ACCUPLACER objectives containing wording similar to the following is to assess mathematics in situations involving either real-world problem solving or problem solving in a mathematical context.

- A.3.a Rate problems including ratio and proportion
- A.3.b Percent problems
- A.3.d Measurement problems