Content of the Long-Term Trend Assessments Compared to Main NAEP

A reaction paper prepared for the National Assessment Governing Board

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February 2017

In December 2016, Edward Haertel from Stanford University prepared a white paper for the National Assessment Governing Board (Governing Board) on the future of the National Assessment of Educational Progress (NAEP) Long-Term Trend (LTT) assessments. The purpose of the white paper was to inform the Governing Board's deliberations as to whether LTT assessments should be continued independently from main NAEP assessments, whether it is feasible to blend LTT assessments with main NAEP assessment, and related questions. This paper delves more deeply into the issue of the considerable differences in content between the LTT assessments developed in the 1970s and 1980s and today's "gold standard" main NAEP. [2]

Reasons for Starting the LTTs

Beginning in the mid-1980s, NAEP made a large shift into what at the time was considered the modern era. As explained by Haertel, in 1984 NAEP moved from the Education Commission of the States to Educational Testing Service (ETS) under a new design and reporting approach based on item response theory (IRT) scales. Although not mentioned by Haertel, work began in 1988 on substantially updating the reading and mathematics frameworks for the Trial State Assessments (TSAs) to be inaugurated in 1990 and 1992. Considering changes in the design, reporting, and especially in the frameworks, it became clear that the existing item pools with vestiges dating back to the 1970s could not support the demands of the new frameworks and reporting goals. So, NAEP began two different data collections: 1) a scaled-back version of NAEP as it was in the 1980s to maintain comparability to the past ("Long-Term Trend" [LTT]), and 2) a modern NAEP based on new frameworks and innovative items as well as a new design and new procedures for data collection, analysis, and reporting ("main NAEP").

The TSAs had a profound impact on the content of NAEP. Anticipating the 1988 legislation authorizing the state-by-state assessments, in mid-1987 the federal government arranged for a special grant from the National Science Foundation and the Department of Education to the Council of Chief State School Officers (CCSSO). The grant was for the CCSSO to conduct a National Assessment Planning Project that had the primary responsibility for recommending objectives for the state-level mathematics assessment in 1990 (CCSSO, 1988).

The CCSSO's National Assessment Planning Project had a steering committee that included policymakers, practitioners, and citizens nominated by 18 national organizations, and a mathematics objectives committee comprised of mathematics educators from various states, mathematicians, parents, and citizens. According to the *1990 Mathematics Objectives*, the mathematics objectives committee did consider maintaining some of the content of prior assessments to allow reporting trends

[1] The author was involved in aspects of NAEP assessment development at Education Commission of the States from 1974 to 1983, and at Educational Testing Service from 1984 to 1994.

[2] The author expresses grateful thanks to the reviewers that contributed to improving this paper.

in performance. However, the committee also changed the framework in important ways. They decided that mathematics is far more holistic than implied by the content-by-process matrix used for the previous assessments, and organized the 1990 framework according to mathematical abilities and content areas with fewer, broader areas. They wanted to develop a forward-thinking assessment that could lead instruction, so they placed more emphasis on problem-solving, as well as geometry and algebra, and less on numbers. They also introduced a new calculator use policy for all three grades, and constructed-response questions designed to provide an extended view of students' mathematical abilities.

During this same period, Congress created the National Assessment Governing Board, giving it responsibility for formulating policy for NAEP including developing assessment objectives. Thus, in late 1989 and early 1990 the National Assessment Governing Board carried out the process of developing the updated reading framework for the continuation of the Trial State Assessments in 1992. To help with the *1992 Reading Framework*, the Governing Board awarded a contract to the CCSSO. The consensus development process involved a steering committee of members from 16 national organizations and a 15-member planning committee consisting of experts in reading. According to the *1992 Reading Framework*, the development guidelines included accounting for contemporary research on reading and expanding the range of assessment tools to new approaches and formats. Decisions about the framework incorporated substantial current research about the characteristics of good readers, and that reading is a dynamic, complex interaction among the reader, the text, and the context of any reading situation. The framework reflected these considerations by assessing reading for literary experience, reading to be informed, and reading to perform a task. The framework adopted the view of reading as an interactive, integrated process of constructing, extending, and examining meaning. Also, the Governing Board supported including many constructed-response items and selecting authentic texts drawn from materials used by students in real, everyday reading.

In contrast to the auspicious start of main NAEP in 1990 and its high quality since then, the LTTs began as subsets of previous NAEP assessments of reading and mathematics reported in the 1980s. The idea was to maintain a link to the past while moving into the future with main NAEP.

The LTTs were not planned to be carried forward indefinitely and have a varied history which often is overlooked:

- **1970s to 1980s—changing item pools**. From the first mathematics and reading assessments in the 1970s and early 1980s at the Education Commission of the States (ECS) through the 1980s at ETS, NAEP used a system of updating objectives with each cycle together with a policy of releasing items to the public and developing new items to respond to the updated objectives. Thus, by 1990 the item pools had changed to some extent over time but still had some "old" items. Haertel observed as part of his paper that it was surprisingly difficult to answer the question of what the LTT assessments in reading and mathematics actually assess.
- **1990s to present—fewer changes in the items pools**. During the 1990s, the subsets of LTT items were not changed. However, as noted by Haertel the LTT trend lines were disrupted in 2004 when various changes were made, including new testing accommodation policies that brought the LTT into conformity with the requirements of the individuals with Disabilities Education Act of 1990 and other legislation. (p.2) Updating the LTTs continued in 2008 although the 2004 scales were maintained.

Because of their history, the LTTs have no explicit frameworks, so as observed by Haertel, it is surprisingly difficult to answer the question of what the LTT assessments in reading and mathematics actually assess. As explained by Haertel:

> Rather, the LTTs began as collections of exercises,[3] operationalizing lists of objectives that changed over time. Some fraction of those items survived being released (i.e., were kept secure for reuse) and also survived screening on technical criteria, screening for bias, screening for outdated or obsolete content, or elimination on any other grounds. These surviving exercises, sometimes revised, augmented with some additional exercises intended to measure the same content, became the LTT exercise pools. (p.20)

It is important to recognize that the LTTs have **not** kept continually administering the exact same items over and over again since 1971 as believed by some people (see Haertel p.19). The LTT blocks began life in 1990 as subsets of NAEP assessments given in the 1980s that had been developed in accordance with objectives that had changed over time since the 1970s. The initial 1990 LTTs were readministered several times and then updated in 2004 and 2008. The most recent administration of the LTTs was in 2011-12.

Why the LTTs Are of Questionable Validity in Today's World

The LTTs are subsets of the NAEP reading and mathematics assessments that were widely viewed by educators in the late 1980s as assessing outdated views of reading and mathematics, so much so, that considerable energy was devoted to updating the content of the NAEP assessments for the Trial State Assessments. With advances in research during the past 20 years, the original LTT views have become further antiquated and can be expected to become increasingly so into the future:

- In reading, to be considered appropriate for the TSAs in 1992, the NAEP reading framework was updated to include an integrated view of reading comprehension. The International Reading Association had passed a resolution on assessment at its 1988 annual conference, stating, in part: "RESOLVED that…reading assessments reflect recent advances in the understanding of the reading process… [and that] assessment measures defining reading as sequence of discrete skills be discouraged."
- In mathematics, drawing on a report of "Issues in the Field" and *NCTM's Curriculum and Evaluation Standards* (1987), the NAEP mathematics framework was updated to be more holistic, to emphasize problem-solving (30 percent at all three grades), and to include more geometry and algebra, as well as calculators.

The LTTs are subsets of assessments developed according to different measurement and reporting approaches than are typical today. Arguably, the LTT items and assessments do not conform to current best practices:

- Because each item measured a relatively specific objective, there was a view that each item told its own story. Considerable energy was spent in providing reports of individual items together with the percentages of correct answers. Today, we are more confident about results based on *reliable* measures of constructs involving robust sets of items.

[3] In the 1970s, NAEP items were referred to as "exercises."

- To compare across ages, exactly the same items had to be used at ages 9 and 13, ages 13 and 17, and ages 9, 13, and 17. This meant 9-year-olds needed to be able to answer two portions of the items administered at age 13 (those at ages 9 and 13, and at 9, 13, and 17), and similarly 13-year-olds had to be able to answer two portions of the items given at age 17. This put a ceiling effect on the difficulty of the items that is particularly noticeable at age 17. In 1984, NAEP began using item response theory (IRT) scaling to enhance the comparability of results across ages, groups, and time, because estimated achievement based on IRT scaling is not dependent on specific items.
- The reading LTTs are comprised almost wholly of multiple-choice items, and the mathematics LTTs are 75 to 80 percent multiple-choice. Today's main NAEP assessments are at least half constructed response, if not more.
- The reading LTTs include passages developed specifically for NAEP, a practice that is not considered appropriate for main NAEP. Main NAEP uses authentic texts actually found and used by students in real, everyday reading.

There is considerable confusion about what the LTTs actually assess. Despite various claims on National Center for Education Statistics (NCES) websites, in blogs, or in publications, the LTT reading and mathematics assessments were not developed according to explicit content frameworks, or designed to measure particular sets of knowledge and skills:

- As noted in Haertel's paper (p.20), it is a widely held belief that the LTTs have tested the same items for more than three decades and so have a unique ability to track changes. However, the reality is that the LTTs have changed. The NAEP assessments underpinning the LTTs changed considerably in the 1970s and 80s, and, subsequently, the LTTs have kept changing even though much more slowly.
- All available descriptions of LTT frameworks are post hoc and therefore, subject to interpretation (including those reproduced in this paper). However, not all accounts of the LTT assessments are careful about describing them. For example, the NCES website used in part of Haertel's paper (p.22) (https://nces.ed.gov/nationsreportcard/**ltt**/**more**about.aspx) says that the LTT "uses substantially the same assessments decade after decade," and lists what the mathematics and reading LTTs were designed to measure.

Summary of Findings About LTT Content

This section summarizes the results of research into the content of the LTT assessments. The last two sections provide details about the aspects of content addressed in the research, first for reading and then for mathematics. In most cases, the content of the LTTs is compared to main NAEP to highlight the extent of the differences.

READING

The following summarizes the comparisons between the main NAEP and LTT reading assessments:

- Main NAEP reading assessments are based on a well-defined comprehensive framework that was developed using a legislatively defined process. According to the *2015 Abridged Reading Framework*, NAEP's definition of reading is grounded in scientific research. The **LTT** assessments are not based on frameworks. They are based on recreating blocks to replicate the same text

types with the same item formats at approximately the same level of difficulty as the released parent blocks developed in the 1970s and 80s.

- As described in the framework, main NAEP selects high-quality literary and informational material from authentic sources. **LTT** contains various disparate materials written for the assessment, primarily short expository pieces and documents. There is a category called "other" for riddles, visuals, and sentence matches.
- The average word length of main NAEP reading passages is 840 at grade 4, 924 at grade 8, and 1,174 at grade 12. These averages exceed the maximum length of the "long" passages in **LTT**, and most of the **LTT** passages are less than 150 words.
- At grade 4, students in main NAEP spend about half the assessment time responding to multiple-choice questions and the other half responding to constructed-response questions. Students in grades 8 and 12 spend a greater amount of time on constructed-response questions. In contrast, the **LTT** reading assessments are nearly all multiple-choice (92 to 95 percent).
- From 70 to 80 percent of the items in main NAEP are developed to measure higher-order reading cognitive behaviors (interpret and integrate, evaluate and critique). In comparison, the majority of the items in **LTT** assess retrieving basic information.
- Main NAEP includes a systematic assessment of vocabulary knowledge, and **LTT** does not.

By today's rigorous standards set by main NAEP as well as by the Progress in International Reading Literacy Study (PIRLS) and the Programme for International Student Assessment (PISA) international assessments, the passages and items in the LTT reading assessments are unlikely to be considered valid and robust assessments of reading. The LTTs assess straightforward comprehension of short pieces of text that are not authentic in the world of 2017, but are carefully replicated to retain their dated features. Reading comprehension is assessed almost wholly by multiple-choice questions. The LTT assessments will become increasingly irrelevant as students perform greater amounts of their reading online, and reading assessments move into the digital age.

MATHEMATICS

The following summarizes the comparisons for mathematics:

- Main NAEP's content areas describe an integrated view of mathematics that begins with a solid grasp of whole numbers and builds through measurement, geometry, statistics, and algebra to a firm foundation for learning calculus. In contrast, **LTT** has more content areas but there is little evidence of progression or integration in the objectives. At ages 9 and 13, more than half the items are devoted to numbers/numeration with about an equal smattering of variables/relationships, shape/size/position, measurement, and probability/statistics. The age 17 items are repeats of age 13. There is little content appropriate for age 17.
- Main NAEP ensures that students have a range of items across three levels of mathematical complexity—low, medium, and high. The **LTT** has an emphasis on "bare" computation.
- Any items resembling the high-complexity items in main NAEP seem to be missing entirely from the **LTT** examples that were available.
- In main NAEP, half the assessment time is devoted to constructed-response questions, including those requiring extended responses where students can demonstrate their thinking and problem-solving skills. The majority of the **LLT** items are multiple-choice and the rest are short answer (e.g., one or two digit numerical answers).

- Main NAEP has a rigorous review process to ensure the item content is appropriate to today's environment. In contrast, the **LTT** items appear to have much more emphasis on the metric system and vocabulary generally than the curriculum has today, especially at age 9.

The mathematics assessments on which LTT is based focused on assessing growth in the **same** mathematics from age 9 to 13, age 13 to 17, and age 9 to 13 to 17, leaving little relevant only to age 17. The LTTs emphasize knowledge and skill much more than problem-solving, making them essentially basic skills assessments, with some of the content outdated.

Comparing the Content of Main NAEP Reading Assessments (2009-2015) with the LTT Reading Assessments (1990-2012)

The detailed look at the content of the reading LTTs is focused on the LTTs since the 1990s, because as explained in the introduction to this paper, that is about when the reading LTTs came into existence. Since then, the LTTs have been modified somewhat to provide accommodations to students with disabilities and English language learners, but the knowledge and skills remain essentially the same.

Gloria Dion, director of NAEP Test Development at Educational Testing Service (ETS) explained: "While there is a document of broad objectives from 1983-84, there is no framework or specifications document for the LTT Reading Assessment" (personal communication, December 2016).[4] Since the first LTT assessments, there have been some reconfigurations and some blocks released in 2004 and 2008. However, Dion continued, "the development process for new blocks has been a close replication of the types of texts and items in the original parent instrument. That is, the new LTT reading blocks are developed to assess the same text types with same item format at approximately the same level of difficulty as the released parent block at each age." (personal communication, December 2016)

To highlight the differences between the content of today's state-of-art NAEP reading assessments and the field prior to 1990, characteristics of today's main NAEP are provided followed by information about the reading LTTs.

Definition of Reading

Main NAEP. According to the *2015 Abridged Reading Framework* published by the National Assessment Governing Board (National Assessment Governing Board, 2015a), the reading framework presents the assessment's conceptual base and discusses its content. The framework has not been changed since 2009 and applies to the assessments between 2009 and 2015 (the previous framework was used from 1992 through 2007).

The main NAEP reading assessments are guided by a definition of reading that reflects scientific research, draws on multiple sources, and conceptualizes reading as an active and complex process that involves:

- Understanding written text
- Developing and interpreting meaning
- Using meaning as appropriate to the type of text, purpose, and situation

[4] The author is very grateful to Gloria Dion, Patricia Donahue, and Shannon Richards of ETS for providing invaluable information about the LTTs and main NAEP.

LTT. As explained above, while there are NAEP objectives from 1983-84 (an assessment containing items in the LTTs) there is no framework or specifications document for the LTT reading assessment. The LTT most likely also contains items from previous NAEP assessments. Backtracking—the *1983-84 Reading Objectives* (NAEP, 1984) were based on the 1979-80 objectives combining reading and literature, which were preceded by two sets of reading objectives in 1970 and 1975 as well as two sets of literature objectives in 1970 and 1975. Perhaps because of integrating reading and literature, the 1983-84 objectives had a more fragmented view of reading than today's NAEP, including two separate objectives that dealt with comprehension—comprehension (basic understanding, primarily of expository passages) and extends comprehension (deliberate, conscious analyses, primarily of literary passages).

Types of Texts

Main NAEP. According to the *2015 Abridged Reading Framework*, reading passages are selected to represent high-quality literary and informational material, such that many NAEP passages require interpretive and critical reading skills. The assessments include two types of texts:

- Literary texts—fiction, literary non-fiction (e.g., essays, speeches, biographies), and poetry. Several aspects of text structures and features, as well as literary techniques, may be assessed for all grades. At grade 4, this includes problem conflict, figurative language, cause and effect, point of view, diction and word choice, and organizational patterns in poetry such as verse and stanza, along with the basic elements of rhyme scheme, rhythm, mood, themes, and intent. These components become increasingly sophisticated as students move through elementary, middle, and high school grades. For example, grade 12 includes dramatic irony, denotation and connotation, rhetorical devices, high levels of abstraction, and complex poetry arrangements.
- Informational texts—exposition, argumentation and persuasive texts, and procedural text and documents. At grade 4, this includes textbook passages, news articles, encyclopedia entries, sequences, point of view, evidence, compare and contrast, and procedural text supplementary to continuous text. These are represented in grade 8 and 12 with increasing complexity. Some examples at grade 12 include social commentary essays, historical accounts, persuasive brochures and advertisements, manuals, and contracts.

Examples of texts included in the *2015 Abridged Reading Framework* were: grade 4—*Little Great White* by Pamela S. Turner, an expository text about how scientists care for a white shark in captivity; grade 8—*FUN* by Suzanne Britt Jordan, a literary nonfiction text about the concept of fun; and grade 12—Theodore Roosevelt's 1905 inaugural address about the duties and responsibilities of being president.

Further information about the number of main NAEP texts and their lengths was obtained from the work of two expert panels convened by the National Center for Education Statistics (NCES): *A Comparison of the PIRLS 2011 and NAEP 2011 Fourth Grade Reading Assessments* and *Comparison of the PISA 2009 and NAEP 2009 Reading Assessments.*

Table 1 shows that the main NAEP assessments have 12 passages at grade 4; and 16 and 17 passages at grades 8 and 12. At grades 4 and 8, the passages are evenly divided between literary and informative, but grade 12 includes more informative passages. Passages (or pairs of passages) are presented in 25-minute blocks, followed by about 9-11 items at grades 8 and 12.

Table 1. NAEP Passage Text Types

	Literary	Informative
Grade 4 (2011)	7	5
Grade 8 (2009)	8	8
Grade 12 (2009)	4	13

Notes: At grade 4, the literary passages included four fiction, one non-fiction, and two poems (NCES: NAEP and PIRLS comparison). At grade 8, passages were evenly distributed, including two poems; at grade 12, about one-quarter were literary (including one poem) and about three-quarters were informational "to mirror the distribution of the kinds of texts students encounter as they progress through the education system" (NCES: NAEP and PISA comparison).

LTT. The 1983-84 NAEP objectives do not describe the types of texts to use in the assessments, but they do call for comprehending various types of written materials and reading for a variety of purposes. The examples of texts included shopping lists, complex essays, literary works, science textbooks, historical essays, mail-order catalogs, instructions to assemble a bicycle, research reports, a play, dictionaries, encyclopedias, bibliographies, and abstracts.

NAEP has classified the texts in the LTT reading assessments in three categories: expository, narrative, and document/other (for riddles, visuals, and sentence matches). There are 10 15-minute blocks at each age—9, 13, and 17. The number of texts per block ranges from three to six. A substantial portion of the blocks are common across ages: 9 and 13, 13 and 17, and ages 9, 13, and 17. The total number of items in 2012 was 88 at age 9, 106 at age 13, and 103 at age 17.

Table 2 shows the distribution of items according to text type (P. Donahue, personal communication, January 2017). Combining the expository and document/other categories, the LTTs basically assess informational reading at all three ages. This is different than the relatively equal distribution of literary and information texts in main NAEP at grades 4 and 8, although there is a better match at grade 12.

Table 2. Distribution of LTT Reading Items by Text Type

	Age 9		Age 13		Age 17	
Text Type	Number	Percentage	Number	Percentage	Number	Percentage
Expository	58	66%	63	59%	73	70%
Narrative	21	24%	19	18%	13	13%
Document/Other	9	10%	24	23%	17	17%

The LTT passages from the 1990 LTTs on the NAEP Questions Tool website, half released in 2004 and half in 2008, are still representative of the LTT reading assessments as updated after 2004 and 2008 (P. Donahue, personal communication, January 2017). The website shows about five passages per age, about half of which overlap grades. Taken together, the passages represent a variety of written materials and purposes. Some topics appear dated (e.g., paper routes, tall tales) or are unsuitable for a broad range of students (e.g., poem asking mother for dog). There are a disproportionate number of passages about history (e.g., frontier women, elephant seal hunting, starting work as a teenager in the 1900s, women getting to vote), probably because historical topics have less likelihood of becoming further dated.

Length of Reading Passages

Main NAEP. Table 3 shows the length of the main NAEP reading passages. The expert panels comparing NAEP to PIRLS and PISA, respectively, both mentioned that NAEP had the longest passages.

Table 3. NAEP Reading Passage Word Counts

	Grade 4 (2011)	Grade 8 (2009)	Grade 12 (2009)
Average number of words	840 (721 with poetry)	923.6	1173.5
Range of words in passages	47-1,147	219-1,429	771-1,429

Notes: At grade 4, the NAEP passages (including pairs of passages) were longer than the PIRLS passages with the exception of NAEP's two poems (47 words and 197 words).

LTT. Table 4 shows general specifications for the LTT reading assessments, which indicates the lengths of LTT reading passages (P. Donahue, personal communication, January 2017). At all three grades, the LTT passages are substantially shorter than those in main NAEP. Also, with shorter passages and shorter blocks, there are fewer items per passage.

Table 4. General Specifications for Lengths and Number of LTT Reading Items per Passage (based on LTT passages in the past)

	Long Passages		Short Paragraphs and Poems	
	Number of Words	Number of Items	Number of Words	Number of Items
Age 9	250-500	3-6 MC or 2MC w/ 1 long answer	50-125	1-2 MC
Age 13	250-625	3-6 MC or 2-3MC w/ 1 long answer	50-125	1-2 MC
Age 17	250-800	same	50-150	1-2 MC

Notes: "Other" tasks include several illustrations at each age. For example, cereal boxes, directions, and snowman at age 9, each with one multiple-choice question; telephone bill, traffic ticket, coupon, advertisement, and table at ages 13 and 17, each with two to three multiple-choice questions at age 13 and three multiple-choice questions at age 17.

Passage Difficulty

Main NAEP. Table 5 provides information about the difficulty of the passages in main NAEP. Several readability formulas were used to compare passage difficulty between NAEP and PIRLS at the 4th grade (the three shown in the table and the Fry Graph with 6.9 average grade level) and NAEP and PISA at the 8th and 12th grades (the three shown in the table and the FORCAST Formula for non-continuous text). The measures in table 5 were selected to summarize comparisons across grades on measures of continuous text (rather than forms and graphics such as used in PISA).

It should be noted that the expert panel comparing NAEP and PIRLS cautioned that readability formulas are used as a quick assessment of the difficulty of text and do not account for certain features of the text that also have influences on reading comprehension, such as text structure, topic, and appeal.

Table 5. Average Readability Levels of NAEP Reading Passages

Readability Measure	Grade 4 (2011)	Grade 8 (2009)	Grade 12 (2009)
Flesch Reading Ease Score	Fairly easy (score 76.5)	Standard (score 69.4)	Standard (score 62.4)
Flesch-Kincaid Grade Level	5.9	7.5	8.8
Lexile	Sixth grade (score 910)		
Dale-Chall Grade Level Formula		6.9 Average (range 5.2 - 8.3)	7.4 Average (range 5.4 - 9.4)

Notes from NCES expert panel reviews:
The Flesch Reading Ease measure is based on the number of words, syllables, and sentences in adult reading materials.
The Flesch-Kincaid Grade Level is most reliable when used to assess upper elementary and secondary materials. It is based on the number of words, syllables, and sentences in a text, but with different weighting than the Flesch Reading Ease measure.
The Lexile analysis takes into account sentence length and word frequency.
The Dale-Chall Formula uses a familiar words list common to students and rates the text against it as well as the text's total number of words and sentences.

LTT. There did not seem to be any information available on the difficulty levels of the written materials in the LTT reading assessments. However, given the overlaps across ages and the short passages, it seems likely that the LTT texts would have lower reading difficulty than main NAEP, which already may be low at grade 8 and especially at grade 12.

Item Response Mode

Main NAEP. At grade 4, students spend about half of the assessment time responding to multiple-choice questions and the other half responding to constructed-response questions. Students in grades 8 and 12 spend a greater amount of time on constructed-response questions.

LTT. Table 6 shows that the LTT reading assessments are comprised almost wholly of multiple-choice items.

Table 6. Percentages of Multiple-Choice and Constructed-Response Items in the NAEP LTT Reading Assessments

Item Format	Age 9	Age 13	Age 17
Multiple-Choice	95%	93%	92%
Constructed Response	5%	7%	8%

Cognitive Targets

Main NAEP. NAEP addresses three sets of cognitive reading behaviors, with 30 percent of the items at grade 4 asking students to locate and recall information, but only 20 percent at grades 8 and 12. At all three grades, about half the NAEP reading assessment items require students to integrate and interpret the information they have read—50 percent at grades 4 and 8, and 45 percent at grade 12. As much as another one-third of the assessment asks students to evaluate the quality of the texts—20 percent at grade 4, 30 percent at grade 8, and 35 percent of grade 12. This means from 70 to 80 percent of the items should assess higher-level cognitive targets. Considering both the passages and the items, the NCES expert panel comparing NAEP and PIRLS concluded that "the NAEP 2011 reading assessment may be more cognitively challenging than PIRLS 2011 for U.S. fourth-grade students" (NCES, 2011, p.17).

- Locate and recall—identify main ideas and supporting details, essential elements of a story.
- Integrate and interpret—compare and contrast, examine relationships across different parts of texts or multiple texts, process information logically and completely, relate texts to their own experiences.
- Critique and evaluate—view text objectively, assess text critically from numerous perspectives and synthesize it with other texts and experiences, judge the effectiveness of specific textual features.

LTT. According to the NAEP Questions Tool, the LTT items have the following content classifications:

- Comprehends What Is Read
- Analyzes What Has Been Read
- Interprets What Has Been Read
- Evaluates What Has Been Read (only at ages 13 and 17)

These classifications may have been derived from the *1983-84 Reading Objectives*. In that document, "Comprehends What Is Read" is the first major objective and "Extends Comprehension" is the second major objective, but the second objective has three sub-objectives: "Analyzes," "Interprets," and "Evaluates What Has Been Read."

Haertel's paper contained a different classification of the LTT reading items. Although accompanied by a footnote that NCES descriptions of LTT were sometimes inconsistent, Haertel reported the following about the LTTs from the NCES website previously cited in this paper:

"The NAEP long-term trend reading assessment… was designed to measure students' ability to

- Locate specific information in text provided,
- Make inferences across a passage to provide an explanation, and
- Identify the main idea in the text."

Reinforcing the point about inconsistency of information about LTT, ETS has different classifications for the LTT reading items: identify main idea, locate information verbatim, locate information, and inference (G. Dion, personal communication, December 2016).

Despite the alternative sources about classifications, the documentation in the NAEP Questions Tool and looking at the released items suggests that the LTT items primarily assess information retrieval. However, there are only 19 released items at age 9, 19 released items at age 13, and 16 released items at age 17, with many overlap items.

Vocabulary Assessment

Main NAEP. *The 2009 Reading Framework* introduced a new systematic assessment of vocabulary knowledge. The vocabulary assessment involves the interpretation of words in the context of a passage. The vocabulary items function both as a measure of passage comprehension and as a test of specific knowledge of a word's meaning. A sufficient number of vocabulary questions at each grade provide reliable and valid information about students' vocabulary knowledge.

LTT. The LTTs may have some isolated items asking about vocabulary. However, systematic assessment of vocabulary knowledge is not part of the LTTs.

Comparing the Content of Main NAEP Mathematics Assessments (2009-2015) with the LTT Mathematics Assessments (1990-2012)

The detailed look at the content of the mathematics LTTs is focused on the LTTs since the 1990s, because as explained in the introduction to this paper, that is about when the mathematics LTTs came into existence. Since then, the 2004 and 2008 LTTs were modified somewhat to provide accommodations to students with disabilities and English language learners but the knowledge and skills remain essentially the same (Institute of Education Sciences, National Center for Education Statistics, 2009).

Mathematics Content Domains

Main NAEP. According the *2015 Abridged Mathematics Framework* published by the National Assessment Governing Board (National Assessment Governing Board, 2015b), the framework applies to the 2009-2015 assessments. In 2009, modifications were made to introduce clarifications at grades 4 and 8, and new content objectives were introduced at grade 12 to identify the essential mathematics knowledge and skills required for college and workplace training.

The 2015 Mathematics Framework specifies that NAEP assessment questions measure one of five mathematical content areas:

- Number properties and operations—including computation and understanding of number concepts
- Measurement—including use of instruments, application of processes, and concepts of area and volume
- Geometry—including spatial reasoning and applying geometric properties
- Data analysis, statistics, and probability—including graphical displays and statistics
- Algebra—including representations and relationships

The following summarizes the grade expectations for each of the content areas, beginning with **number properties and operations**. At grade 4, students should have a solid grasp of whole numbers and begin to understand fractions. They should be able to identify place values, and add, subtract, multiply, and divide whole numbers. At grade 8, they should be comfortable with decimals, percentages, and common fractions, and be able to solve problems involving proportionality and rates. They should have some familiarity with naturally occurring irrational numbers (e.g., square roots, pi). By grade 12, students should be able to establish the validity of numerical properties using mathematical arguments.

Measurement focuses on length (perimeter, distance, and height) at grade 4; areas and angles at grade 8, and volumes and rates (e.g., speed) at grade 12. **Geometry** has developed into the study of the possible structures of space. At grade 4, this includes figures in the plane (lines, circles, triangles, rectangles, and squares) and in space (cubes, sphere, and cylinders). At grade 8, there is an emphasis on understanding properties of figures, such as parallelism, perpendicularity, and angle relations, and a mixing with measurement. By grade 12, geometry and algebra merge to provide the basis for calculus. In **data analysis, statistics, and probability,** students at grade 4 should be able to compare two data sets and understand the basic concepts of chance. By grade 8, they should be able to use data organizing and summarizing techniques, analyze statistical claims, and make statistical inferences; such that by grade 12, they should be able to use a wide variety of statistical techniques, including fitting models to data.

Algebra at grade 4 emphasizes extending numerical patterns and the idea of unknown quantities. By grade 8 students should be familiar with linear functions, and by grade 12 students should be familiar with nonlinear functions as well as with expressions involving several variables, systems of linear equations, and solutions to inequalities.

Table 7 shows the operational distribution of items across the content areas in the 2013 Mathematics Assessment (S. Richards, personal communication, January 2017). At grade 4, the assessment emphasizes number properties and operations (40 percent), but that is not the case at grades 8 and 12. Grade 8 emphasizes algebra (30 percent) the most, with 15 to 19 percent in each of the other areas. At grade 12, more than half the assessment is in two areas—algebra (32 percent) and data analysis, statistics, and probability (24 percent).

Table 7. 2013 NAEP Mathematics Distribution of Items by Content Area

Content Area	Grade 4	Grade 8	Grade 12
Number Properties and Operations	40% (60)	19% (29)	12% (22)
Measurement	18% (27)	19% (29)	12% (22)
Geometry	15% (22)	17% (26)	19% (37)
Data Analysis, Statistics, and Probability	13% (19)	15% (23)	24% (46)
Algebra	15% (22)	30% (46)	32% (62)
Total	150	153	191

LTT. The beginning chapters of the *1981-82 Mathematics Objectives* published by the Education Commission of the States (National Assessment of Educational Progress, 1981) provide a brief history of the first three mathematics assessments. As described by Haertel, a three-dimensional classification scheme was used to categorize the mathematics objectives for the first assessment during the 1972-73 school year: uses of mathematics, content (17 areas), and objectives and abilities. Haertel observes that "almost half a century later, some of these early objectives seem quite dated, and others seem out of place in a mathematics assessment." This is not surprising, since in developing the 1972-73 objectives, they were compared with other statements in mathematics education literature and found to be consistent with objectives appearing in the preceding 25 years (back to 1947). About half the items from the first assessment were released, but the other half were retained for measuring trends in the second assessment during 1977-78 and beyond.

The objectives for the second mathematics assessment were changed considerably and organized into a content-by-process matrix. This content-by-process matrix also was used for the third assessment in 1981-82, and also updated for the fourth assessment in 1985-86*. So, it makes some sense that NAEP uses the content domains from the* 1981-82 Mathematics Objectives to describe the LTT trend assessment.

The content domains from the 1981-82 objectives included:

- Numbers and numeration
- Variables and relationships
- Shape, size and position

- Measurement
- Probability and statistics
- Technology (not in LTT)

Courtesy of ETS, table 8 contains the distribution of the current mathematics LTT according to the content described in the *1981-82 Mathematics Objectives* (S. Richards, personal communication, January 2017). The items are contained in six 15-minute blocks, with half of them overlapping ages—one each for 9 and 13, 13 and 17, and 9, 13, and 17. There are 137 items at age 9, 157 at age 13, and 157 at age 17. Considering that the total assessment time is 90 minutes, this does not provide very much time for each item (about 30 seconds). These time limitations suggest short, factual questions. The majority of the items at ages 9 and 13 assess numbers and numeration, as do 42 percent at age 17. At age 17, more emphasis on numbers (42 percent) than algebra (27 percent) is very different from main NAEP. Also, there is little attention to probability and statistics and no attention the more up-to-date content related to data analyses.

Table 8. 2012 LTT Mathematics Operational Blocks by Content Area

Content Area	**Age 9**	**Age 13**	**Age 17**
Numbers and Numeration	53% (72)	52% (82)	42% (67)
Variables and Relationships	11% (15)	11% (18)	27% (43)
Shape, Size, and Position	9% (12)	11% (18)	11% (17)
Measurement	15% (20)	14% (22)	12% (19)
Probability and Statistics	13% (18)	11% (17)	7% (11)
Total	137	157	157

Information about the actual content of the LTT mathematics items was obtained from the NAEP Questions Tool, which contains 40 released items at age 9, 43 released items at age 13, and 53 released items at age 17 (https://nces.ed.gov/NationsReportCard/nqt/Search/SearchOptions). Additionally, ETS forwarded "*Appendix 1: Mathematics Content Objectives by Age Level*" which contained "item descriptions of the current long-term trend items" matched to the 1981-82 objectives. Unfortunately, appendix 1 is only a copy of appendix 1 from an unknown publication with an unknown date that has been passed forward in a file drawer. Nevertheless, this is additional "public" documentation and the author is grateful to have it. Appendix A to this document contains the information in the mysterious appendix, which lists descriptions for 54 items at age 9, 76 at age 13, and 51 at age 17.

Looking across the LTT trend items in the NAEP Questions Tool together with the item descriptions in appendix A reveals information about the content of LTTs (even though some questions overlap). At age 9, consistent with the distribution across the content areas, the majority of the items assess numbers and numeration, and the majority are multiple-choice. Nearly all the short constructed-response items require single- or double-digit numerical answers. There are items about place value and as many as 20 items requiring basic computation. There appears to be a disproportionate number of items requiring symbol and vocabulary recognition, especially concerning the metric system (e.g., >, X, =, prime number, kiloliter, pentagon, circumference, degree, fifths, kilogram, kilometer). There are several number sentences, and an item about the property of a square. There also are more than a dozen items involving reading tallies, tables, charts, and graphs, and one on the concept of probability. The items are not classified according to process domain categories. However, very few would be considered

moderate-complexity items and none high-complexity items, even though according to ETS 31 percent (42) are classified as application and problem-solving (see table 10).

At age 13, again, the majority of the items assess numbers and numeration, including decimals, fractions, and percentages. There are a few that would seem unusual in light of today's curriculum (e.g., two on improper fractions, and the meaning of 0.7 percent). There are about half a dozen examples in variables and relationships, including several algebraic expressions but also several logic statement items. Fewer than 10 examples are in geometry, and only a couple in probability and statistics. Again, most of the items are multiple-choice and they are not classified according to process. There is very little problem-solving.

At age 17, about half the items assess number and numeration, including decimals, fractions, and percentages similar to age 13. The next largest group of items assesses variables and relationships, consistent with the distribution across the content areas shown in table 8. However, almost all of the items are the same as those at age 13. One of the few "hard" questions, described as "reason about an algebraic equation" was: If P/41= 64, what does P/82 equal? Ans:32. The same as at ages 9 and 13, most are multiple-choice, and the process classifications are not provided. However, among the released questions and the item descriptions, there are hardly any items that could be considered for age 17 only. This is a serious concern, especially considering that the high-complexity types of items are nonexistent.

Process Domains

Main NAEP. The framework for main NAEP does not have cognitive processes. It uses a hierarchical model called levels of mathematical complexity, with low, moderate, and high as three levels.

- Low-complexity questions require students to recall or recognize concepts or procedures specified in the framework.
- Moderate-complexity questions involve more flexible thinking and choice among alternatives. The student needs to decide what to do and how to do it.
- High-complexity questions make heavy demands on students to use reasoning, planning analysis, judgement, and creative thought. Students may need to justify mathematical statements, construct a mathematical argument, or generalize from specific examples.

Table 9 shows the operational distribution of items by mathematical complexity for the 2013 mathematics assessment (S. Richards, personal communication, January 2017). The majority of mathematics items in main NAEP are considered to have low mathematical complexity. Most of the rest have moderate complexity, and about 5 percent high complexity.

Table 9. 2013 NAEP Mathematics Distribution of Items by Cognitive Complexity

Mathematical Complexity	Grade 4	Grade 8	Grade 12
Low	58% (87)	56% (86)	55% (106)
Moderate	36% (54)	39% (60)	39% (75)
High	6% (9)	5% (7)	5% (10)

LTT. The process domain for the objectives in the 1981-82 Mathematics Assessment had five categories, with each category suggesting a mental process.

- Mathematical knowledge—refers to the recall and recognition of mathematical ideas. It ordinarily relies on the memory process.
- Mathematical skill—refers to the routine manipulation of mathematical ideas and relies on algorithmic processes that are standard procedures leading to answers.
- Mathematical understanding—refers to the explanation and interpretation of mathematical knowledge and relies primarily on translation processes.
- Mathematical application and problem-solving—refer to the use of mathematical knowledge, skill and understanding in solving both routine and nonroutine problems. These items require a sequence of processes; reasoning and decision-making processes must be used.
- Attitudes toward mathematics (not in the LTTs)

Table 10 shows how the LTT items have been distributed across the process categories (G. Dion, personal communication, December 2016). However, care should be taken in interpreting the data in this table. As observed above, there are very few, if any, problem-solving items contained in the released items or in the item descriptions. It is possible that the bulk of items placed in the application and problem-solving category are application rather than problem-solving.

Table 10. 2012 LTT Mathematics Operational Blocks by Process Domain

Process Domain	Age 9	Age 13	Age 17
Mathematical Knowledge	23% (32)	15% (23)	15% (24)
Mathematical Skill	35% (48)	48% (76)	46% (73)
Mathematical Understanding	11% (15)	16% (25)	16% (25)
Mathematical Application and Problem-Solving	31% (42)	21% (33)	22% (35)
Total	137	157	Total

Question Formats

Main NAEP. Testing time on NAEP is divided evenly between multiple-choice questions and two types of constructed-response questions. Short constructed-response questions require students to give either a numerical result or the correct name or classification for a group of mathematical objects, draw an example of given concept, or possibly write a brief explanation for a given result. Responses can be scored correct/incorrect or partially correct. Extended constructed-response questions require students to consider a situation that demands more than a numerical or short response. For example, the student may be asked to describe a situation, analyze a graph or table of data, or set up and solve an equation given a real-world problem.

LTT. The LTTs use two types of questions—multiple-choice and short constructed response. Table 11 shows the operational distribution of item types in the 2012 LTT mathematics assessments (S. Richards, personal communication, January 2017). From 74 to 80 percent of the items are multiple-choice. In contrast to main NAEP, there are even more multiple-choice questions at age 17.

Table 11: 2012 LTT Mathematics Operational Blocks by Item Type

Item Type	Age 9	Age 13	Age 17
Multiple-Choice	74% (102)	76% (120)	80% (126)
Constructed Response	26% (35)	24% (37)	20% (31)
Total	137	157	157

Calculator Use

Main NAEP. About two-thirds of the assessment blocks at each grade contain questions for which calculators are not allowed. The other one-third of the blocks allow calculator use and contain some questions that would be difficult to solve without a calculator.

- Grade 4: A four-function calculator is supplied to students
- Grade 8: A scientific calculator is supplied to students
- Grade 12: Students are allowed to bring whatever calculator, graphing or other, they are accustomed to using in the classroom, with some restrictions for test security purposes. For students who do not bring a calculator to use on the assessment, NAEP will provide a scientific calculator. Having a graphing calculator is not an advantage in answering the NAEP questions.

LTT. The LTT mathematics assessments do not allow calculators, although some early NAEP mathematics assessments did consider calculator use. According to Mary Lindquist, the 1977-78 assessment even included a special booklet to conduct a calculator study. At that point in time, however, students had to be trained on how to use the calculator and they often took longer with the calculator (M. Lindquist, personal communication, January 2017).[5]

This accentuates the differences between then and now, with the move toward digital assessments of mathematics in NAEP and the states in the United States, and internationally in the Trends in International Mathematics and Science Study (TIMSS) and PISA.

Conclusion

Haertel summarizes a number of arguments against maintaining the LTTs in their current form, with the last reason being the most important (p.23):

- It is expensive.
- Maintaining two trends is confusing.
- That performance on outdated content is no longer of interest to policymakers or other stakeholders.
- A range of changes in schooling, in assessment technology, and in society at large are rendering the results irrelevant and possibly invalid.

Looking carefully at the content of the LTTs, including the released items in the NAEP Questions Tool, calls the quality and relevancy of the LTTs into question. In today's reach for higher education standards as presented by the National Assessment Governing Board, how students performed on yesterday's

[5] Mary Lindquist, past President of the National Council of the Teachers of Mathematics, was an ECS mathematics consultant for the 1977-78, 1981-82, and 1985-86 NAEP assessments, and participated in CCSSO's 1990 National Assessment Planning Project as a member of the mathematics objectives committee.

assessment may not be particularly relevant, especially compared to the results on state-of-the-art main NAEP.

Additionally, there are indications that people do not understand what the LTTs are, including the seriously erroneous view that the LTTs have been giving the same assessment items since 1971. As a more appropriate view, Haertel's paper includes two graphics from the Executive Summary of the 2012 NAEP Long-Term Trend report showing a break in the LTT trends in reading (NCES, 2013). There are solid lines back to 2004 when the new accommodation procedures were implemented, and then broken lines showing some issues with trend accuracy back to 1971.

Taking concerns about the LTTs' quality into consideration together with the various misunderstandings about what they actually are measuring and for how long raises concern about their validity. In times of scarce funding resources, the expense of fielding LTTs must be considered in view of other priorities. For example, the Governing Board has been conducting extensive research into whether NAEP can be a valid indicator of college and career readiness (National Assessment Governing Board, 2013 and 2016). If tradeoffs need to be made, it makes more sense to extend NAEP into areas more in tune with the future than the past, and move NAEP forward toward its goal of helping our 8th and 12th grade students be better academically prepared for education and job training after high school.

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Appendix A

Item Descriptions for LTT Long-Term Trend Mathematics

Numbers and Numeration

Number Concepts (whole numbers, fractions, decimals, percentages, and integers)

Age 9: Apply place value (2), Identify greatest money value (2), Identify greatest number (1), Relate part to whole (1), Understand place value (3)

Age 13: Change decimal to percent (1), Convert decimal to percent (1), Convert fraction to decimal (3), Identify greatest number (1), Identify number line property (1), Write improper fraction (1), Understand concept of percent (1), Understand decimal place value (1), Understand percent less than 1 (1)

Age 17: Convert decimal to fraction (3), Convert percent to decimal (1), Identify number line property (1), Understand concept of percent (1), Understand decimal place value (2), Understand opposite of an integer (2), Use concept of percent (1)

Operations (whole numbers, fractions, decimals, percentages, and integers)

Age 9: Add whole numbers (4), Divide whole numbers (2), Multiply whole numbers (2), Subtract whole numbers (3), Apply multiplication (1), Apply operation of addition (1), Apply operation of subtraction (1)

Age 13: Add whole numbers (3), Add integers (1), Divide integers (2), Subtract whole numbers (6), Find percent given numbers (2), Find percent greater than 100 (2), Find percent of number (1), Apply operation of addition (1)

Age 17: Add integers (2), Divide integers (2), Multiply fractions (3), Subtract decimals (1), Find number given percent (1), Find percent given numbers (1) Find percent greater than 100 (2), Find percent of number (2), Identify sign of divisor (1), Simplify square root (1)

Estimation

Age 9: Estimate large number (1)

Age 13: Estimate cost of pencils (1), Estimate cost using percent (1), Estimate total cost (1)

Age 17: Estimate cost of pencils (1), Estimate cost using percent (1), Estimate square root (2), Estimate total cost (1)

Properties

Age 9: Use property of transitivity (1)

Age 13: Apply transitive property (1), Identify even number property (2), Use property of transitivity (1)

Variables and Relationships

Relations

Age 13: Find common factor (1), Identify even number (1)

Use of Variables

Age 9: Translate words into numbers (1), Write multiplication sentence (1)

Age 13: Add monomials (1), Identify algebraic identify (1), Identify number sentence (1), Write addition sentence (1)

Age 17: Add monomials (1), Apply concept of inequality (1), Define equivalent equations (1), Identify algebraic identity (1), Identify linear inequality (1), Multiply equation by a constant (1)

Functions and Formulas

Age 13: Evaluate algebraic expression (1)

Age 17: Evaluate algebraic expression (1), Evaluate function for value (1)

Operations with Variables

Age 9: Solve number sentence (2)

Age 13: Solve number sentence (1)

Age 17: Solve number sentence (1)

Shape, Size, and Position

Recognition of Figures

Age 9: Apply property of square

Age 13: Identify a sphere (1), Identify parallelograms (1)

Age 17: Identify a sphere (1), Relate circle to square

Definitions, postulates, and theorems

Age 13: Identify parallel lines (1), Identify perpendicular lines (1), Apply supplementary angles (1), Apply vertical angles (1), Apply triangle inequality (1)

Age 17: Apply angle addition property (1), Identify parallel lines (1), Identify perpendicular lines (1), Apply supplementary angles (1), Apply vertical angles (1), Use properties of triangles (4)

Measurement

Units/Estimation of Measurements

Age 9: Estimate weight (metric) (1), Identify greatest metric unit (1)

Age 13: Convert metric unites (2), Estimate difference in length (1), Estimate height of door (1), Estimate total weight (1), Identify greatest metric unit (1), Identify unit of length (1), Identify unit of weight (1)

Age 17: Estimate circumference (1), Estimate difference in length (1), Estimate height of door (1), Estimate total weight (1), Estimate weight (1), Relate meter to yard (1)

Instrument Reading

Age 9: Read scale (1)

Age 13: Read length using ruler (1), Use ruler to measure length (1)

Area, Perimeter, and Volume

Age 9: Determine amount of change (1), Determine distance on a map (1), Find area of rectangle (2), Find perimeter of rectangle (2)

Age 13: Find area of rectangle (2), Find area of square (1), Find perimeter of rectangle (2)

Age 17: Find area given perimeter (1), Find area of irregular shape (1), Find area of rectangle (1)

Probability and Statistics

Organizing, displaying, and interpreting information (tallies, tables charts, and graphs)

Age 9: Interpret data in bar graph (1), Interpret data in circle graph (1), Interpret data in table (1), Interpret tally chart (1), Read circle graph (1), Read data from table (1), Read data in bar graph (1), Read tally chart (2), Compute using data in table (1), Compute with data in bar graph (1)

Age 13: Interpret data in bar graph (1), Interpret data in circle graph (1), Interpret data in table (1), Read circle graph (1), Read data from table (1), Read data in bar graph (1), Compute using data in table (1), Compute with data in bar graph (1)

Age 17: Interpret data in table (2), Interpret line graph (1), Read line graph (1), Compute using data in table (2)

Probability (simple, compound and independent events, odds)

Age 9: Apply concept of probability (1)

Age 13: Identify expected value (1), Understand probability (1),

Age 17: Identify expected value (1)

Note: At the time appendix 1 was developed, the LTTs also included some items from mathematical methods and discrete mathematics, areas introduced in 1985-86. More specifically, note that at the time appendix 1 was developed, the LTTs also included some items from mathematical methods and discrete mathematics, areas introduced in 1985-86: age 9 (seven items), age 13 (five items), and age 17 (eight items).